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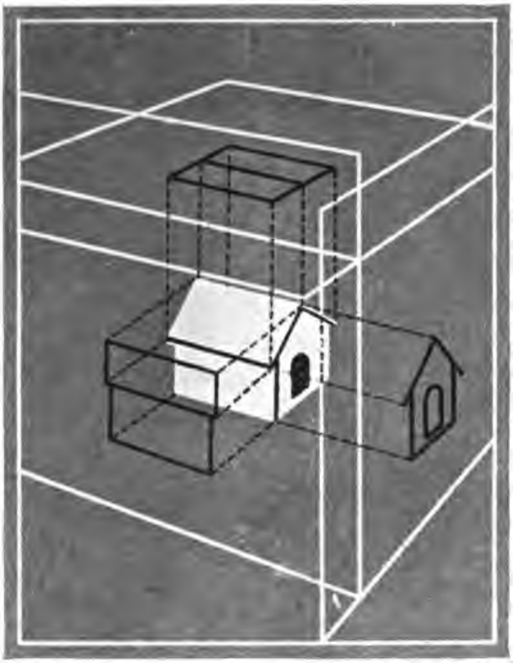

FIG. 1.—THE PLANES OF PROJECTION

[*Frontispiece*]

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To Professor Denison
With the Compliments of the Author.

DESCRIPTIVE GEOMETRY

FOR

STUDENTS OF ENGINEERING

BY

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Society of Mechanical Engineers; Society of Civil Engineers;
Society for the Promotion of Engineering Education; etc.*

THIRD EDITION
FIRST THOUSAND

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JAMES AMBROSE MOYER

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PREFACE.

THIS book is the result of teaching descriptive geometry to students of engineering. My aim is to present the subject so as to make it most easily applicable to the requirements of recent engineering practice. The methods of presentation in this book, therefore, are not traditional. Experience has shown that most students in our best technical schools have difficulty in applying their knowledge of this subject to subsequent work in structural and machine design. Two things have been attempted in this book to overcome this failure of our students: (1) The notation is essentially the same as that used in mechanical drawing. For a long time, practical drafting and descriptive geometry have had too little in common. (2) The exercises have been carefully graded to encourage a student to do thinking for himself; and, to stimulate his interest, many concrete exercises, showing usually practical applications, have been inserted. Such exercises, I think, should be introduced from the beginning, so that the student may see the practical application of his problems as he goes along.

The data for the exercises are stated by the system of coördinates used in analytic geometry. Reasons for choosing this system are obvious. For a class beginning this subject, there is a great advantage in stating the exercises with absolute definiteness. If a definite problem is not given, many students, in order to show a satisfactory solution, will waste much time selecting data; and others will present drawings that for their complication are mostly unintelligible.

Illustrations are of more use than much wordy description. For this reason an unusually large number of perspective and orthographic drawings have been inserted. The illustrations in perspective are very helpful. Whenever it is possible, however, students should be encouraged to make models of cardboard and pencils that they may "build" what they are drawing.

This book is not intended for self-instruction. Like languages, this subject can be learned successfully only from a teacher, and not alone from books and lectures. The student must take the time to work out many exercises. Space has been left on the right-hand pages for lecture notes and sketches. The student may well put the solutions for many of the exercises on these pages.

A good deal of space is taken to explain Problems 6, 7, and 8. These are considered fundamental; and the teacher should be sure they are mastered before the student goes further. With these problems well in mind, there should be no difficulty with those that follow. It has been my object to make the explanations of the problems throughout the book consistently briefer as the subject-matter is developed.

I am under great obligation to Professor Ira N. Hollis and Professor Lewis J. Johnson for much assistance and encouragement in preparing this book. I owe special acknowledgment, however, to Professor Henry S. Jacoby, who led in teaching this subject with its practical applications. He has carefully read much of this book, and I have received many suggestions from him.

For assistance in many ways I wish to thank my brother, Mr. J. Clarence Moyer, M.E., of Philadelphia, Mr. C. B. Lewis of Cincinnati, and Mr. Bryant White of Cambridge.

J. A. MOYER.

CAMBRIDGE, December, 1903.

PREFACE TO THE SECOND EDITION.

THE gratifying results with the first edition showed that the methods of this book were appreciated beyond my expectations. In the second edition I have added a number of new exercises. Many of these appear throughout the text.

In preparing the second edition the help of Mr. A. E. Norton, Ph.B., has been invaluable to me. For valuable suggestions and criticisms I am much indebted to Commander Barton, U. S. Naval Academy; Prof. Adams, Mass. Inst. of Technology; Prof. Kennedy, Harvard Univ.; Prof. Ogden, Cornell Univ.; Prof. Randall, Brown Univ.; Prof. Spangler, Univ. of Pennsylvania; Prof. Tilden, Univ. of Michigan; Prof. Tracy, Yale Univ.; and Mr. W. V. Moses of the General Electric Company.

The American Bridge Company and the Boston Bridge Works have kindly supplied drawings from which the data for some of the exercises have been taken.

I am much gratified that in Prof. Ferris's book on descriptive geometry which has just been published, an effort is shown to meet, in a degree, practical requirements. Since the first edition of this book appeared, I have received many letters regarding the relative importance to be given this subject from a practical viewpoint in a course in engineering. These inquiries interest me much, and in replying I have gladly given the results of my experience.

J. A. MOYER.

CAMBRIDGE, January, 1905.

PREFACE TO THE THIRD EDITION.

INDUSTRIAL education is becoming, every day, more important in all systems of teaching. The tendency in education is toward the economic applications. The advantages of teaching with the help of practical problems and exercises is more appreciated than ever, with correspondingly more satisfactory results. These new requirements are measured, in a degree, by the success of this book.

In this edition some changes, mostly suggested by teachers, have been made in the text, and an index has been added to make the book more convenient for reference. For very valuable criticisms I am especially indebted to Prof. Dr. Linsel of Berlin, Germany, and Prof. Jacoby of Ithaca. Much of the work of revision has fallen to my colleague Mr. A. E. Norton of Cambridge, whose services I cannot too highly appreciate.

J. A. MOYER.

LYNN, December, 1906.

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DESCRIPTIVE GEOMETRY

INTRODUCTION

1. **Descriptive Geometry** treats of the methods of making drawings to represent objects with mathematical accuracy. There are two common methods for such representation. By one method, called **perspective drawing,** the chief purpose is to produce a picture which will be plain to a person unfamiliar with the methods used for technical drawings. By the other method, however, the chief aim is to show an object with the true dimensions that are needed in the construction of buildings and machines. The drawings are then made by a method which does not give a pictorial effect; but, on the other hand, shows views of the object, from which, by very simple processes, true dimensions of all parts can be quickly obtained. This latter method is called **orthographic projection.** It is the method with which the student must become most familiar, and with which this treatise must most concern him for some time. Perspective drawing will be discussed later. The method of orthographic projection represents the outlines of the object as they might be traced on transparent planes placed around the object as shown in Figs. 1 (frontispiece) and 2*a*, where three views of a hexagonal pyramid are shown pictorially on horizontal and vertical planes. Drawings representing these views by the orthographic method are made in the same way as in mechanical drawing. The object is thus represented as though the eye were infinitely distant; that is, the vanishing of the lines of the object in the distance is not represented.

CHAPTER I

ELEMENTARY PRINCIPLES AND NOTATION

2. The horizontal and two vertical planes upon which the three views of the pyramid are shown in Fig. 2*a* are called the **planes of projection.** These planes are always taken at right angles to each other and are designated according to their position as **horizontal, front vertical,** and **side vertical.** The lines of intersection of the horizontal with the front vertical and side vertical planes are called respectively the X and Y axes. The intersection of the front vertical and side vertical planes is called the Z axis. These axes are shown plainly in the figure, and the point where they intersect is called the **origin,** and is usually marked O.

3. In Fig. 2*a* the planes of projection are shown in a pictorial drawing, where they are placed around a pyramid which is the object to be represented. The planes are arranged as we must imagine them placed to show the top, front, and side views of the pyramid, according to the conventional methods used in practical drafting. In this figure the views of the pyramid shown on the planes of projection, are its outlines made by rays of light reflected from points on the pyramid perpendicular to a plane of projection. The points where the rays pierce these planes are called the **projections** of points on the surface of the pyramid. Thus, in the figure, two corners of the pyramid are marked a and b. From these points dotted lines are drawn representing rays of light reflected from them **perpendicular to the planes of projection.** The intersections of these dotted lines from a and b with the planes are marked respectively a^h, a^f, and a^s and b^h, b^f, and b^s. Of these, the first three are called the **projections of the point** a; and the last three, the **projections of the point** b. The projections of other points are found in the same

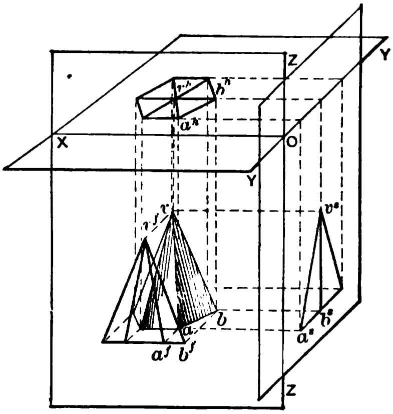
Z
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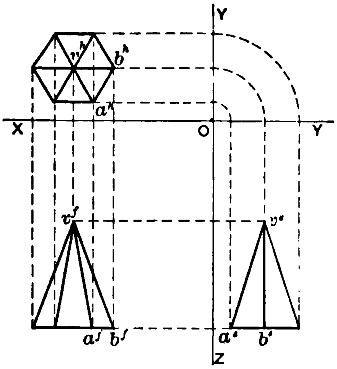
FIG. 2*a*.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Z

FIG. 2*b*.

way, so that the complete projections of the outlines of the pyramid can be made on each of the planes of projection. The complete projections of the outlines of an object are called its **horizontal, front vertical,** and **side vertical projections.** For objects with angular outlines, only the projections of the corners are usually needed. The views of the object are then, of course, drawn by joining the projections of the corners. The dotted lines in the figure, representing the rays from the points on the pyramid to the projections, are called **projecting lines.**

In Fig. 2*a* we see the planes of projection, the hexagonal pyramid, and the projections of the outlines of the pyramid on the planes, as in a picture. All is shown as one complete view, such as we see when several sides of an object we are looking at are seen from a single viewpoint. If, however, the eye is moved so that we see only one projection at a time, a set of three different views would be obtained, called **orthographic projections.** By the methods of orthographic projection, these views would be arranged as in Fig. 2*b*. In this latter figure, a horizontal line is first drawn to represent the *X* axis, usually near the middle of the space to be taken for the drawing. Then at right angles to this line the *Y* and *Z* axes are drawn as shown. To represent by this method the views shown pictorially in Fig. 2*a*, the horizontal projection is drawn *behind* the *X* axis and on the left-hand side of the vertical *Y* axis. The front vertical projection is drawn below the *X* axis and vertically below the horizontal projection. The side vertical projection is also drawn below the *X* axis but on the other side of the *Z* axis. As the views are thus shown the horizontal projection shows the outlines of the pyramid when the eye is immediately over the horizontal plane. The front and side vertical projections represent the outlines as they appear when the eye is moved in front of the front and side vertical planes. This is a very convenient way to arrange the projections of an object, and it has the advantage that it is more easily understood by mechanics of ordinary ability, than some other arrangements of the same projections that will be discussed later.

In Fig. 3*a* the same pyramid that is shown in Fig. 2*a* is rep-

resented in a different position with respect to the planes of projection. In Fig. 2*a* the pyramid is placed *below* the horizontal plane and behind the front plane; in Fig. 3*a*, it is placed *above* the horizontal plane and behind the front plane. For this arrangement, the drawing made by orthographic projection is shown in Fig. 3*b*. This method, however, is scarcely ever used, as it crowds the projections too much for ordinary purposes. Observe that in the orthographic projection the front vertical projection is above the *X* axis, and the horizontal projection is behind as before. It thus happens that for this method of arrangement of views, both the horizontal and the vertical projections are shown in orthographic projection on the same side of the *X* axis. In Fig. 4*a*, however, a little better arrangement of views is shown. Here the pyramid is *above* the horizontal plane and in *front* of the front plane. In orthographic projection the same views are shown in Fig. 4*b*. This arrangement of the projections was once much used in practical drafting. It does not crowd the views as in Fig. 3*b*. In Fig. 5*a* the pyramid is *below* the horizontal plane and in *front* of the front plane. Orthographic projections of these views are shown in Fig. 5*b*. This last arrangement of the projections is also likely to produce crowding, and has no significance in practical drafting.

It should be mentioned here that the arrangement of views shown by orthographic projection in Fig. 2*b* is adopted almost exclusively for engineering drawings in the United States. This arrangement is almost universally applied in modern machine drawing. By this method, the views are placed in the most natural positions to suggest a mental picture of the object represented by the drawings; the top projection is at the top of the drawing, and the front view just in front of the top view. The right-hand side view is at the right-hand side of the sheet. The views are thus arranged where they can best suggest a mental picture to a workman.

4. The actual method of making a drawing like Fig. 2*b* by orthographic projection, will now be taken up. It can be most easily explained by showing how the principal points in the drawing

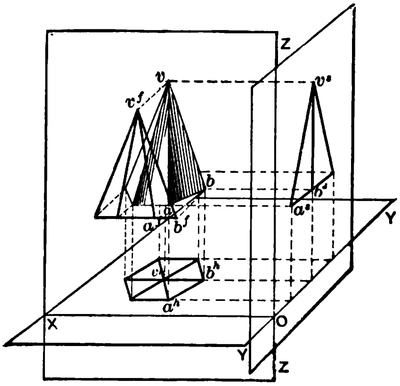

Fig. 3a.

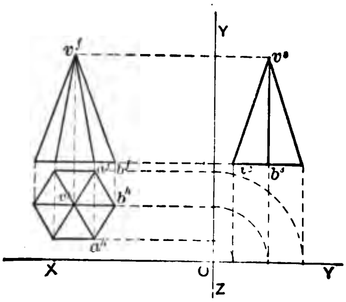

Fig. 3b.

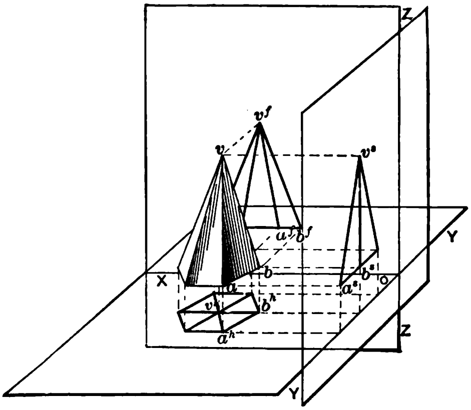

Fig. 4a.

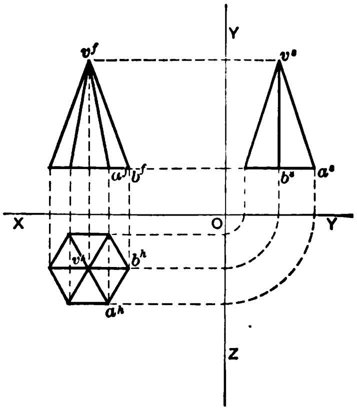

Fig. 4b.

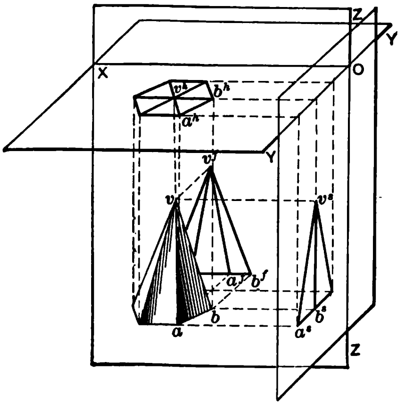

Fig. 5a.

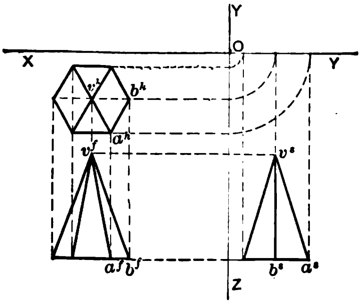

Fig. 5b.

were laid out. A corner of the pyramid marked a has been referred to before; and its projections a^h, a^f, and a^s were pointed out. They are also marked in the orthographic projections in Fig. 2*b*. These projections are located by their distances from the X, Y, and Z axes. Thus the distance from the horizontal projection, a^h, to the vertical Y axis, measured horizontally, shows the distance, in space, from the point a to the side plane. The distance from a^h to the X axis, measured vertically, shows the distance of the point a behind the front vertical plane. The front vertical projection, a^f, is below the X axis, and the distance, measured vertically, from this projection to the axis, shows the distance that the point, a, is below the horizontal plane. The side vertical projection, a^s, is found by laying off a distance, measured horizontally, equal to the distance that the point a is behind the front vertical plane. Of course, the two projections a^f and a^s are at equal distances below the X axis.

5. The methods of **descriptive** or **practical geometry** are useful to engineers and architects in many structural and mechanical operations. In designing and constructing buildings and machines, it is often necessary to find the true size and shape of parts shown on drawings. When the necessary dimension lines are not shown, and when some of the actual lengths are foreshortened, indirect geometrical methods of measurement must be used. The study of this subject is useful, however, for more than its industrial utility. The student becomes accustomed to considering very complicated geometrical combinations, and to follow accurately the correspondence between the drawings and the objects represented. "Descriptive Geometry," a well-known engineer has said, "trains one to see around corners." Indeed it exercises in the most precise manner the power to visualize, which is representing to ourselves, clearly and easily, ideal objects as if they were really before us.

The importance of this subject in training students for work in designing bridges, buildings, and machinery can hardly be overestimated. Nevertheless, **if the study of this subject does not teach the student to think, the goal is missed.**

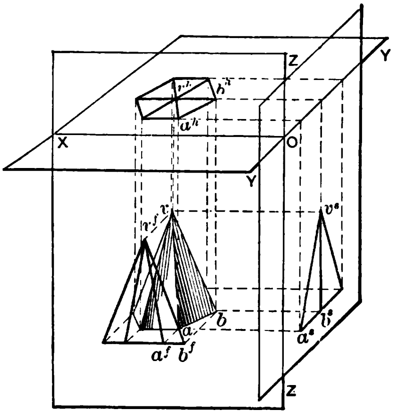

FIG. 2a.

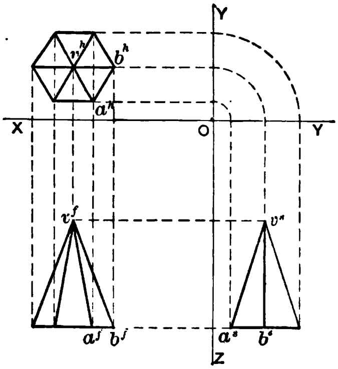

FIG. 2b.

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The methods of descriptive geometry are absolutely general in their application so that, in the work that follows, if the solution of a problem is given for any one of the arrangements of views that have been explained, it is applicable as well for all the others. **The planes of projection,** also, **are indefinite in extent;** that is, they extend without limit in every direction. **Projections of lines may be produced as far as they are needed, when the solution of a problem requires it.**

6. **Planes** other than the planes of projection must be often represented. Any plane that can be shown within the limits of the drawing, will intersect one or more of the planes of projection in straight lines. These lines of intersection, called the **traces** of the plane, are made use of to represent planes in orthographic projection drawings. An oblique plane intersecting the three planes of projection is shown in Fig. 6*a*. The intersection of the plane with the horizontal plane of projection is called its **horizontal trace.** Its intersections with the front and side vertical planes are called respectively its **front vertical** and **side vertical traces.** The same plane is shown in Fig. 6*b* by its traces, as it is represented by the method of orthographic projection. For brevity, however, these traces will be called simply **horizontal, front,** and **side traces.** The simpler name for the last two traces can lead to no confusion and short names are preferable to abbreviations or symbols. For the same reason, the planes of projection will be called, hereafter, simply **horizontal, front,** and **side planes**; and we shall use the corresponding simpler names for the projections of points and lines.

NOTATION.

7. The **projections of a point** in space are designated by a small letter with h, f, or s placed above it to indicate which projection is meant; thus the **horizontal, front,** and **side projections** of a point a are marked respectively a^h, a^f, and a^s. In the drawing, these projections are located by the distances of the point a from the planes of projection, measured parallel to the X, Y, and Z axes;

and in the geometry of space, the distances along these axes are represented respectively by the coördinates x, y, and z.

Distances along the X axis (represented by the x coördinates) are measured to the left or right from the side plane. Long usage has established that these distances shall be considered negative when measured to the left of the side plane and positive when measured to the right.

Distances along the Y axis (represented by the y coördinates) are measured behind or in front of the front plane. These are negative when behind and positive when in front of the front plane.

Distances along the Z axis (represented by the z coördinates) are measured below or above the horizontal plane; negative when below, positive when above.

If a point is represented as: $a\ (-2,\ -3,\ -4)$, we mean that the point a has for its coördinates $x=-2$, $y=-3$, and $z=-4$;—that the point a is 2 units of length to the left of the side plane, 3 units behind the front plane, and 4 units below the horizontal plane.

8. The **projections of a line** are denoted by the projections of two or more points in the line; thus a line containing the two points a and b is called the line $a\,b$ and is represented by the coördinates of these two points.

The point where a line intersects the horizontal plane of projection is marked by the letters $h\,i$; and the intersections of a line with the front and side planes by $f\,i$ and $s\,i$. To locate these points more plainly draw a small circle around their projections as shown in Figs. 8 and 9, on pages 21 and 29.

9. The **traces of planes** are represented in a way suggested by the method of coördinates for points. A plane is shown in Figs. 6*a* and 6*b*. The horizontal and front traces intersect on the X axis to the left of the origin and make with this axis the angles marked α and β. By symbols such a plane, P, is represented as $P\ (x,\ \alpha^\circ,\ \beta^\circ)$. This notation for planes is remembered best by observing that the first number (the x coördinate) represents the

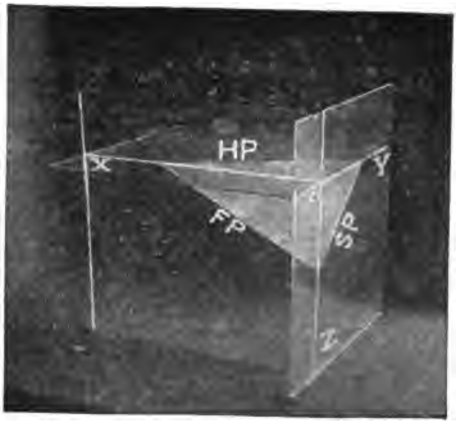

FIG. 6*a*.

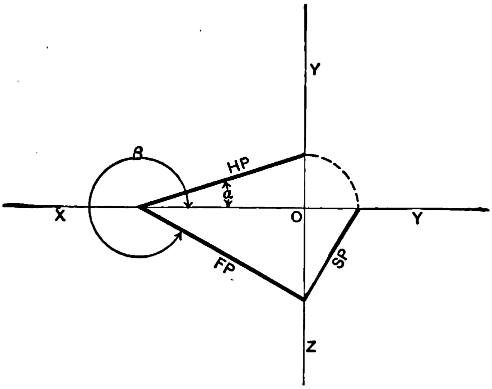

FIG. 6*b*.

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intersection of the traces on the X axis; and that the second and third refer to the number of degrees the horizontal and front traces make with the X axis. These angles are measured in anti-clockwise direction as in trigonometry. In Fig. 6*b* the angles α and β are respectively about 20° and 330°. In such a notation for the traces, it is not necessary to designate the side trace, as any two traces determine the plane, and data concerning a third is usually superfluous. In the drawings, the traces of a plane are marked with the same letter used in *naming* the plane, with *H*, *F*, or *S* written before it. For example, in Fig. 6*b*, the horizontal, front, and side traces of the plane, *P*, are marked respectively *H P*, *F P*, and *S P*.

In all drawings the horizontal and front traces must intersect on the X axis, and testing for this intersection is a check on the accuracy of the constructions.

10. Lines and traces of planes, given or required, are represented in orthographic projections by full lines when visible, by dotted lines when invisible. Projecting lines are indicated by short dashes. Traces of auxiliary planes are usually represented by one long and two short dashes.

CHAPTER II

PROBLEMS RELATING TO THE POINT, LINE, AND PLANE

11. The relation between the actual position of a line in space and its projections on a drawing, is easily conceived for a line that is parallel to a plane of projection. Difficulty is, however, usually experienced in conceiving this relation for a line that is oblique to all the planes of projection. A pictorial drawing of such a line is shown in Fig. 7*a*. The projections of the line are drawn by joining the projections of any two points in the line. For the line shown in the figure, the points most easily determined in the drawing are its intersections with the horizontal and front planes. These intersections are marked a^h and b^f. The former, a^h, has its horizontal projection coincident with the point itself, and its front projection is, of course, in the X axis at a^f. Similarly the latter intersection, b^f, has its horizontal and front projections at b^h (in the X axis) and at b^f. The horizontal and front projections of the line are then drawn by joining a^h with b^h and a^f with b^f.

12. By reversing the process the intersections of the line with the planes of projection can be found when the projections of the line are given; thus in Fig. 7*b* the intersection of the line $a\,b$ with the horizontal plane is the point, a, shown by its horizontal projection, a^h, (behind the intersection of the front projection of the line with the X axis) and its front projection, a^f, (in the X axis). The intersection of the same line with the front plane is the point, b, shown similarly by its projections b^h and b^f.

When only two planes of projection are mentioned (as is the case for most problems), it is assumed the side plane is not needed.

13. Problem 1. *To draw the projections of a line having given the intersections of the line with the horizontal and front planes.*

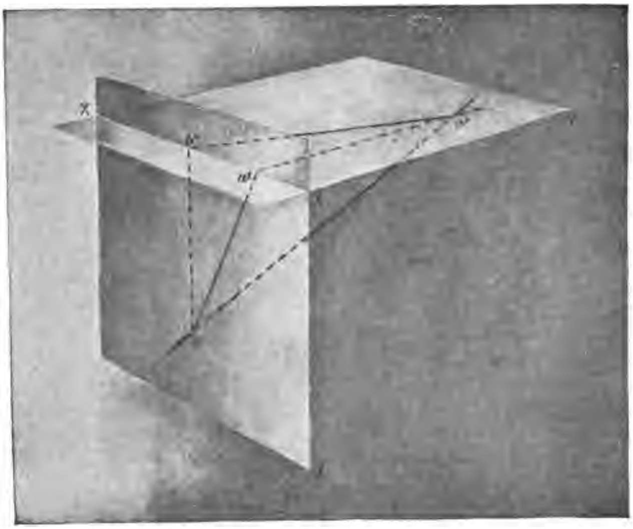

Fig. 7*a*.

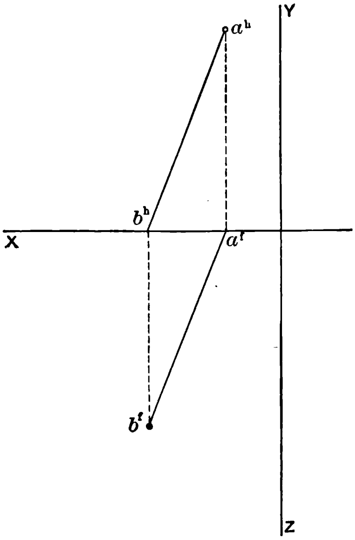

Fig. 7*b*.

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Method. Draw the horizontal projection of the line by joining the horizontal projections of the two intersections. Draw the front projection of the line by joining the front projections of the two intersections.

EXERCISES

For notation see Arts. 7–9.—The coördinates of points are expressed in inches.

1. (*a*) A line intersects the horizontal plane at the point *a* (−3, −2, 0) and the front plane at the point *b* (−1, 0, −2). Draw the projections of the line.
   (*b*) A line passes through the point *e* (−2, −2, −1½) and intersects the horizontal and front planes at *o* (0, 0, 0). Locate the projections of the line.

14. PROBLEM 2.—*Given the projections of a line, to find the point where the line intersects* (*a*) *the horizontal plane,* (*b*) *the front plane.*

Method. For (*a*) produce if necessary the front projection of the line to the X axis. This is the *front* projection of the required point of intersection. To find the *horizontal* projection, draw a perpendicular to the X axis through the front projection just found, to meet the horizontal projection of the given line. For (*b*) produce the horizontal projection of the line to the X axis. This is the *horizontal* projection of the required point of intersection. The *front* projection of the point is then easily found.

EXERCISES

2. Given a line passing through the points *c* (−3, −1, −2) and *d* (−1, −2, −½). Find the points where this line intersects (1) the horizontal plane, (2) the front plane, (3) the side plane.
3. (*a*) Draw the projections of two lines, one parallel to the horizontal plane and oblique to the front plane, the other parallel to the front plane and oblique to the horizontal plane.

(*b*) Find the intersections of each line with the horizontal and front planes of projection.

4. On the line given in Ex. 2, locate a point, *k*, 1 inch below the horizontal plane and another point, *l*, 1¼ inches behind the front plane. Show three projections of each point located.
5. Given the line through the points *e* (−3, −1, −2) and *f* (−1, −2, −1). Locate its projection on the side plane.
6. Assume any line in space, and draw its projections. Find the new projections of this line (*a*) when the *X* axis is perpendicular to its former position (coinciding with the vertical *Y* axis), (*b*) when the *X* axis is revolved 30° (anti-clockwise) from its first position.

   Suggestion. The exercise is easily solved by finding *h i* and *f i* of the line. When the planes of projection are revolved, the positions of these points with respect to the axes will not be changed.

15. A line lying in a given plane intersects the horizontal plane of projection at a point in the horizontal trace; that is, in the line showing the intersection of the given plane with the horizontal plane. The same line intersects also the front plane at a point in the front trace of the given plane. This is illustrated by a pictorial drawing in Fig. 8*a* where a line, *a b*, lying in a plane, *P*, is shown. The line is here produced upward until it intersects the horizontal plane at *h i* and downward to intersect the front plane at *f i*.

The problem now takes this form (see Fig. 8*b*): Given only the **front projection,** $a^f\, b^f$, of the line *a b*, determine the **horizontal projection,** $a^h\, b^h$, so that the line *a b* shall lie in the plane *P*. Instead of finding at once the points $a^h\, b^h$ it is first necessary to locate **both projections** of the **points,** *h i* and *f i*, where the given line *a b* intersects the horizontal and front planes respectively. Then the horizontal projections of the points *h i* and *f i* provide, when joined, the required horizontal projection of the line.

Obviously the **horizontal** projection of the point *h i* is in the **horizontal trace** of **the plane** *P*, and the **front** projection of the same point is in the *X* axis. Similarly the front and horizontal projections of *f i* are respectively

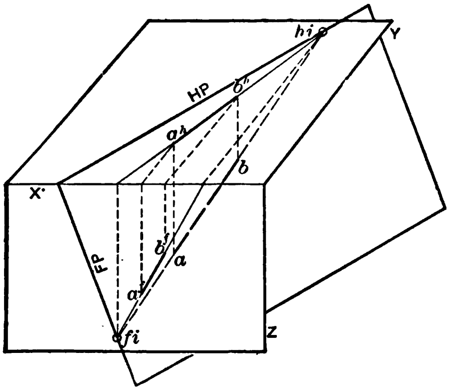

Fig. 8*a*.

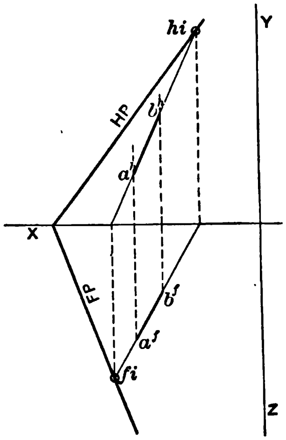

Fig. 8*b*.

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in the front trace of *P* and in the *X* axis. In the figures the projections of *hi* and *fi* in the *X* axis are not marked to avoid confusing the drawings with too many letters.

On the horizontal projection of the line between *hi* and *fi*, thus found, the required projections a^h and b^h are located by projecting lines perpendicular to the *X* axis through a^f and b^f.

16. PROBLEM 3. *Given one projection of a line, or a point, in a given plane, to find the other projection.*

Method. (*a*) Given one projection of a line. Determine the positions of the intersections of the line in the given plane with the horizontal and front planes, and draw the required projection.

Method. (*b*) Given one projection of a point. Draw a projection of *any* line through the given projection of the point. Find the other projection of this line as described above. At the intersection of the projection that has been found, with a projecting line from the given projection of the point, is the required projection.

EXERCISES

7. The line through *a* (−2, −1, ?) and *b* (−1, −¼, ?) is in a plane *P* (0, 150°, 240°). Draw the front and side projections of the line.
8. Given one projection of a line which is oblique to the *X* axis in a plane, *Q*, with traces parallel to this axis. Find another projection of the line.
9. Locate a point, *m*, in the plane, *P*, given in Ex. 7 that shall be ½ inch behind the front plane. In the same plane, locate also a point, *n*, 1¼ inches below the horizontal plane.
10. Draw the three projections of a point, *c*, in a plane, *R*, which is perpendicular to two planes of projection.
11. Draw the three projections of a point, *d*, in a plane, *S*, which is parallel to the horizontal plane.
12. The slope of a roof on a building is represented by its horizontal and front traces, making angles respectively of 150° and 225° with the *X* axis. There is a hole in the roof for a chimney of which the horizontal projection is shown in the drawings as a regular hexagon (length of

sides is $\frac{1}{4}$ inch). The horizontal projection of the center of the hole is located $1\frac{3}{4}$ inches to the left of the intersection of the traces and $\frac{1}{2}$ inch behind the X axis. Draw the front projection of the hole. (The required projection is found readily by drawing the projections of the diagonals of the hexagon.) State the usual notation for the plane of the roof (Art. 9).

13. A circular water pipe, of which the diameter is 1 inch, passes through an inclined floor represented on a drawing by horizontal and front traces making angles respectively of 150° and 210° with the X axis. The center line of the pipe is parallel to the horizontal plane. The front projection of the pipe is therefore a circle, and its center is shown 3 inches to the left of the intersection of the traces and $\frac{7}{8}$ inch below the X axis. Complete the drawing by finding the horizontal projection of the hole through which the pipe passes. (Draw a number of diameters of the circle, producing them when necessary, and find the horizontal projections of these lines. The points in the outline of the horizontal projection of the hole will be vertically over the corresponding points in the given front projection.)

14. The horizontal trace of a plane intersects the X axis at $x = -1\frac{1}{2}$ and makes with it an angle of 120°. A point $o\ (-1,\ -1\frac{1}{2},\ -\frac{1}{2})$ lies in the plane. Determine the front trace of the plane. (Draw through o any line that will have its horizontal intersection ($h\ i$) in the horizontal trace. The $f\ i$ of this line determines the front trace.)

17. PROBLEM 4. *To draw through a given point a line parallel to a given plane.*

Method. Draw one projection of *any line,* and by the method of the last problem, determine its other projection so that the line will lie in the given plane. Through the two projections of the *given point,* draw the projections of the required line parallel to the corresponding projections of the line which has been drawn

in the plane. The line through the point and the line in the plane, shown thus by parallel projections, are parallel to each other; and the line passing through the point, being parallel to a line lying in the plane, is parallel to the plane.

Parallel lines have parallel projections on *any* plane.

EXERCISES

15. Through a point a $(-3,\ -1,\ -1\frac{1}{2})$ draw a line parallel to the plane R $(-3,\ 30°,\ 315°)$.
16. Draw through the point of intersection of the line b $(-3,\ 0,\ -1\frac{1}{2})$, c $(-2\frac{1}{2},\ -1\frac{1}{2},\ -\frac{1}{2})$ with the horizontal plane, a line parallel to the plane T $(0,\ 120°,\ 225°)$.
17. Find the intersection with the side plane, *s i*, of a line with both projections parallel to the X axis.
18. Find the intersection of the line d $(-3,\ -1\frac{1}{2},\ -2\frac{1}{2})$, e $(-1\frac{1}{2},\ -1,\ -1\frac{1}{2})$ with the side plane. Through this point draw the side projection of a line which is to be parallel to the plane U of which the horizontal and front traces are lines parallel to the X axis. The former trace is $1\frac{1}{2}$ inches behind and the latter is 2 inches below the X axis.

18. Remember that the line, *a b*, in Figs. 8*a* and 8*b* which are again shown on the opposite page, is in the plane *P*. Observe, also, that the intersections, *h i* and *f i*, of this line are respectively in the horizontal and front traces of the plane. Now if the projections of a second line in the same plane were given, its intersections, *h i* and *f i*, would be points also in the traces of the plane. Then suppose the conditions of the problem are reversed, and the *projections* of two lines in a plane, *P*, are given and we are to construct the traces, *H P* and *F P*. The horizontal and front intersections, *h i* and *f i*, of each line are usually very easily found. The two horizontal intersections will determine the horizontal trace; and the two front intersections will determine the front trace.

Let us now apply this method to a concrete problem. In Fig. 9 the horizontal and front projections of a triangular pyramid are shown, and we wish to show the plane of a side *a b c* by its traces.

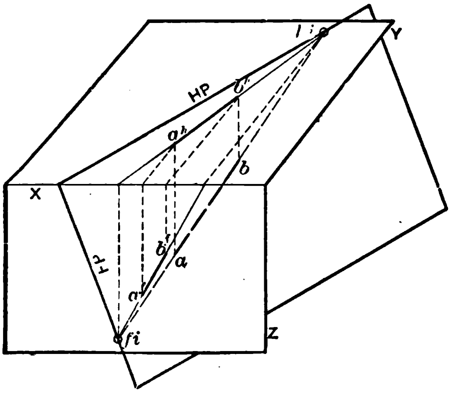

FIG. 8a.

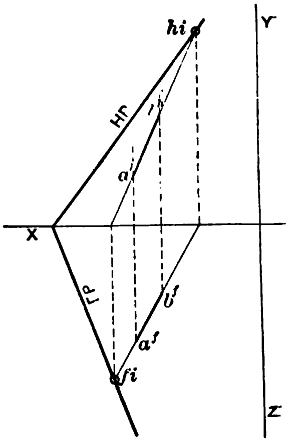

FIG. 8b.

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The projections of two lines *a b* and *a c* in this side are marked in the figure. The horizontal and front intersections of each line with the planes of projection are marked respectively *h i* and *f i*. The horizontal trace of the plane is obtained by joining the two horizontal intersections, and the front trace by joining the two front intersections. If we call the side *a b c* the plane *P*, the horizontal trace should be marked *H P* and the front trace *F P*. The accuracy of the drawing should be checked by observing whether these traces intersect on the *X* axis.

In the example just explained, the lines determining the plane **intersected** * each other; but the solution is the same for **parallel** † **lines.** A plane is determined also by any **three points** not in the same straight line, or by a **point and a line.** The first case, that of the three points, is most easily solved by joining them by straight lines "two and two." In the second case, that of the point and line, solve by drawing through the point *any* line *intersecting* the given line. Both cases are then resolved into that of drawing a plane through **two intersecting lines.**

19. The problems of **drawing a plane through one line parallel to another, or through a given point parallel to two given lines,** are scarcely more than variations of this same problem. In the first case, one line in the required plane is given by its projections and the direction of another. Through any point in the first line, draw a line parallel to the second, and the problem is easily solved. In the last case the directions of two lines are given, and the plane is determined by drawing two lines parallel to them through the given point. Both cases are resolved again into passing a plane through two intersecting lines.

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* *Intersecting lines* meet in a point. If two lines intersect, the horizontal, front, and side projections intersect in the corresponding projections of the point.

† *Parallel lines* have their horizontal, front, and side projections parallel; that is, if two lines are parallel the two horizontal, the two front, and the two side projections are parallel.

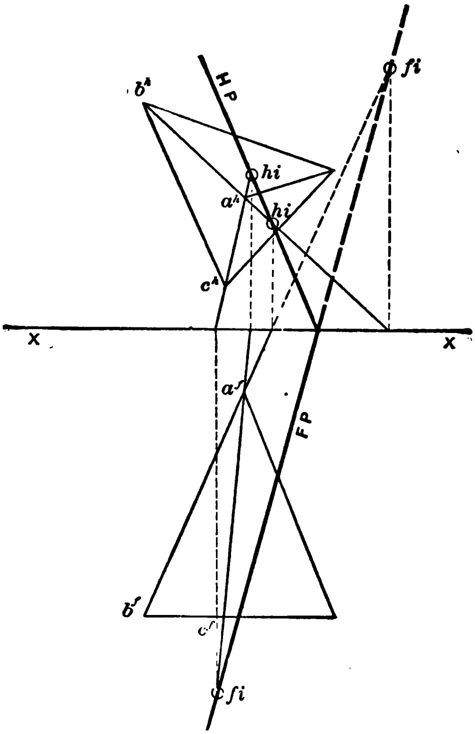


FIG. 9.

20. PROBLEM 5. *To pass a plane through two intersecting or parallel lines.*

Method. Find the points of intersection of the given lines with the horizontal and front planes. The line joining the intersections in the horizontal plane is the horizontal trace of the plane. The line joining the intersections in the front plane is the front trace. The side trace is located by the horizontal and front traces.

EXERCISES

In the following exercises mark the horizontal, front, and side traces of a plane not otherwise designated, by the letters *H P, F P,* **and** *S P.*

19. The line through *a* $(-2, -\frac{1}{2}, -\frac{3}{4})$ and *b* $(-1, -1\frac{1}{2}, -\frac{1}{4})$ intersects the line through *c* $(-2, -1\frac{1}{4}, 0)$ and *d* $(-\frac{1}{2}, -\frac{1}{2}, -1\frac{1}{2})$ at *e* $(-1\frac{1}{2}, -1, -\frac{1}{2})$. Pass a plane through these two lines.

20. Draw the projections of any two lines *intersecting* at the point *m* $(-1\frac{1}{2}, -1, -\frac{1}{2})$. Determine the plane in which these two lines lie.

21. Draw the traces of a plane through the *parallel* lines *a* $(-3\frac{1}{2}, -\frac{1}{4}, -1)$, *b* $(-2\frac{3}{4}, -\frac{3}{4}, -\frac{1}{4})$; and *c* $(-1\frac{1}{4}, -1\frac{1}{4}, -1)$, *d* $(-\frac{1}{2}, -1\frac{3}{4}, -\frac{1}{4})$.

22. Pass a plane through the three points: *a* $(-2, -\frac{1}{4}, -1)$, *b* $(-1, -1, -\frac{1}{4})$, and *c* $(-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{4})$.

23. Find the plane passing through the line *d* $(-1, -\frac{1}{4}, -\frac{3}{4})$, *e* $(-\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2})$ and the point *f* $(-2\frac{3}{4}, -1, +1)$.

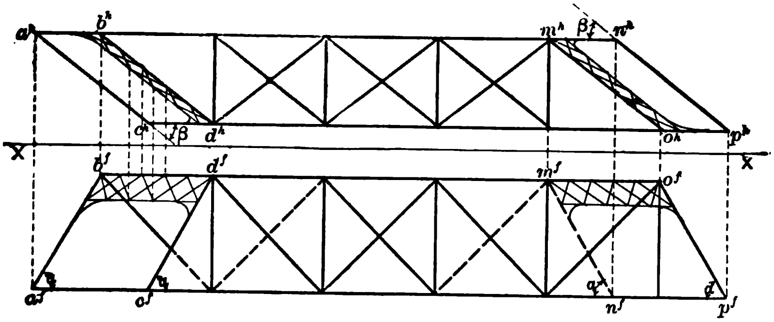
24. (*a*) Through the line *g* $(-1\frac{1}{4}, -\frac{1}{2}, -1)$, *h* $(-\frac{1}{2}, -1, -\frac{1}{4})$ pass a plane parallel to the line *i* $(-2\frac{1}{2}, -1\frac{1}{4}, -\frac{1}{4})$, *j* $(-1\frac{3}{4}, -\frac{1}{2}, -\frac{3}{4})$.

    (*b*) Through *i j* pass a plane parallel to *g h*. (Observe that the traces of the two planes are parallel.)

25. Through the point *o* $(-1, -1\frac{1}{2}, -\frac{1}{2})$ pass a plane parallel to the lines *k* $(-3\frac{1}{4}, -1\frac{1}{4}, -1)$, *l* $(-2\frac{1}{4}, -\frac{1}{4}, -1\frac{1}{2})$; and *m* $(-2\frac{1}{2}, -1, -\frac{1}{2})$, *n* $(-2, -2, -1)$.

26. Pass a plane which is parallel to the X axis through the point p $(-2, -1, -1)$. Draw the trace of the plane on the side plane.

27. A skew bridge is shown in the figure below. The corners of the portal of the bridge are given by the points a $(-4, -2\frac{1}{2}, -3)$, b $(-2\frac{1}{2}, -2\frac{1}{2}, -\frac{3}{4})$, c $(-1\frac{1}{2}, -\frac{1}{2}, -3)$, d $(0, -\frac{1}{2}, -\frac{3}{4})$ as in the figure. The **portal** is the plane bounded by the lines $a\,b$, $c\,d$, and $b\,d$. The lines $a\,b$ and $c\,d$ are called the **end posts** and $b\,d$ is the **portal strut.** Determine the plane of the portal.



28. The end posts of the portal of a skew bridge are shown on an engineer's drawings by the lines r $(-2, -\frac{1}{4}, -\frac{3}{4})$, s $(-1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{8})$; and t $(-1\frac{1}{2}, -\frac{1}{4}, -\frac{3}{4})$, u $(-1, -\frac{3}{4}, -\frac{1}{8})$. The horizontal projection shows the plan and the front projection the elevation. Draw on the three planes of projection the traces of the plane in which the lines $r\,s$ and $t\,u$ lie. Measure in degrees the angle between the plane of the end posts and the horizontal plane.

29. The points a $(-2\frac{1}{2}, -\frac{1}{2}, -1)$, b $(-2, -\frac{3}{4}, -\frac{1}{2})$, c $(-1\frac{1}{2}, -\frac{1}{4}, -1\frac{1}{2})$, and d $(-1, -\frac{1}{2}, -\frac{3}{4})$ determine a surface. Can you find a plane which will include these points? Is it then a plane surface?

30. A triangular pyramid is given by the points a $(-3\frac{1}{2}, -2, -2\frac{1}{2})$, b $(-2, -\frac{1}{2}, -2\frac{1}{2})$, and c $(-1\frac{1}{2}, -2\frac{1}{2}, -2\frac{1}{2})$ at the corners of its base, and its vertex d which is $1\frac{1}{2}$ inches above the base and equidistant from a, b, and c. Find the traces

of the planes of *a d b* and *b d c*, marking them *P* and *Q* respectively.

21. In practice, the problems of descriptive geometry are mainly those of finding the intersections of planes with other planes (Problem 6), the intersections of lines with planes (Problem 7), or the true lengths of lines (Problem 8). These may be spoken of as the fundamental problems of descriptive geometry. Every student must now learn these solutions, so that he can use them immediately under any conditions.

In Fig. 10*a* two planes *P* and *Q* are shown by their traces. A triangular pyramid is also shown by its projections. The plane of its side *a b d* is the plane marked *P*; and another side *b d c* is the plane *Q*. Obviously the intersection of these two sides (planes) is the edge, *d b*, of the pyramid; and no construction is necessary when the points *b* and *d* are given by their projections. The first of the fundamental problems mentioned above arises, however, when the points *b* and *d* are not known, the traces of the planes are given, and the line in which the planes intersect must be found. The problem to be solved is shown more simply in Fig. 10*b*, where the traces *H P*, *F P*, *H Q*, and *F Q* are shown without the pyramid. Explanation is unnecessary to show that if a line in the plane *P* intersects a line in the plane *Q*, **a point in the line of intersection of the two planes is determined.** The horizontal trace, *H P*, is a **line** in the plane *P*; and the horizontal trace *H Q* is a **line** in the plane *Q*. Where *H P* and *H Q* intersect is a point, then, in the required line of intersection of the two planes. The horizontal projection of this point is n^h, and the front projection, n^f, is evidently in the *X* axis. Similarly, where the front traces, *F P* and *F Q*, intersect is o^f, the front projection of another point in the line of intersection. The horizontal projection of this point is o^h in the *X* axis. The horizontal and front projections of the line of intersection are found then by drawing $n^h\,o^h$ and $n^f\,o^f$. If these projections are drawn also in Fig. 10*a*, we can check the accuracy of our work by observing whether *n o* coincides with *d b* as it should.

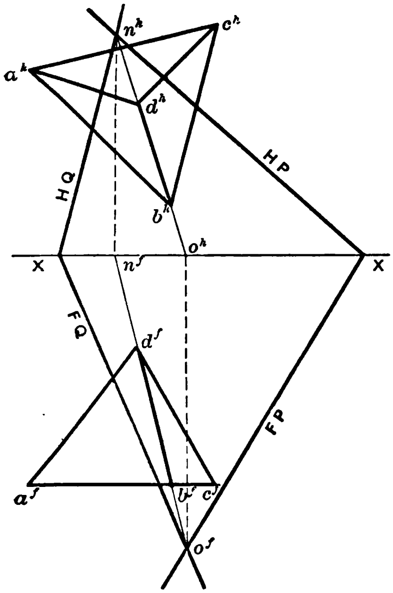


FIG. 10a.

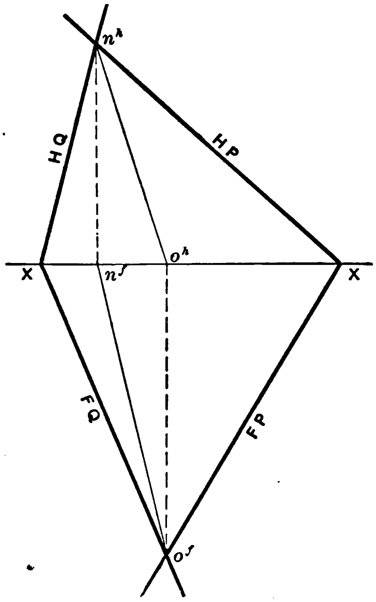


FIG. 10b.

22. In some problems where the line of intersection of two planes is to be found, the traces of the planes do not intersect on the sheet on which the drawing is made. Such a case is shown in Fig. 11, where the front traces do not intersect in the limits of the drawing. For the solution of this problem an auxiliary plane, A, parallel to the front plane of projection is drawn to intersect the planes P and Q. The horizontal trace of this plane is shown by $H A$. This auxiliary plane cuts auxiliary lines from the planes P and Q. The horizontal projections of these lines are, of course, coincident with the trace $H A$, and the front projections are parallel to the front traces. The intersection, o^f, of the front projections of these auxiliary lines determines one point in the line of intersection, and the horizontal projection, o^h, is vertically over o^f in the horizontal projections of both lines. As explained in the preceding paragraph, the point n is also a point in the line of intersection of the planes P and Q. By drawing then $n^h o^h$ and $n^f o^f$ we obtain the two projections of the required line of intersection. If in this problem the horizontal traces did not intersect on the paper, an auxiliary plane would be needed parallel to the horizontal plane.

23. In a case where both planes are parallel to the X axis their line of intersection is likewise parallel to the axis; and the required line is determined by the intersection of the *side* traces of the given planes. In Fig. 12 the planes P and Q are represented by their horizontal and front traces parallel to the X axis. The side traces of the planes are shown at the right on the drawing, intersecting at the point x^s. The horizontal and front *projections* of the required line of intersection are shown by $x^h x^h$ and $x^f x^f$.

24. PROBLEM 6. *To find the line of intersection of two planes.*

Method. The point where the horizontal traces intersect is the point of intersection of the required line with the horizontal plane. The point where the front traces intersect is the point of intersection of the required line with the front plane. After locating the

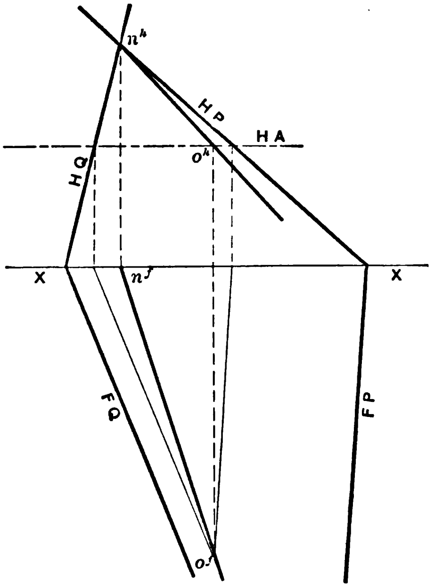


Fig. 11.

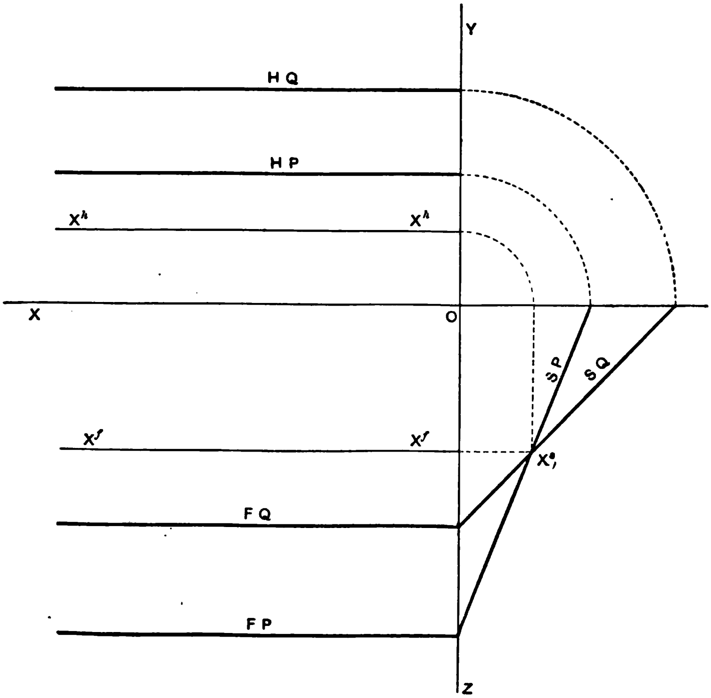


Fig. 12.

two projections of each of these points, join the like projections to obtain the line of intersection.

EXERCISES

31. Find the line of intersection of two planes M (−3, 60°, 300°) and N (−1, 135°, 210°).
32. Draw the horizontal and front projections of the line of intersection of the plane of the end posts in Exercise 28 with another plane, parallel to the X axis, making an angle of 30° with the horizontal plane and passing through a point 1 inch behind the front plane and ¼ inch below the horizontal plane. (Use the side plane.)
33. Find the projections of the intersection of the plane, M, in Exercise 31 with the plane P (−1, 135°, 300°). (Observe that the *line* of intersection is parallel to the front plane and that the front *projection* is parallel to the front traces.)
34. Find the line of intersection of the plane Q (0, 135°, 275°) with a vertical plane parallel to the front plane and 1 inch behind it. (The front projection of the line of intersection is parallel to the front trace of Q. The horiozn― projection is parallel to the X axis.)
35. Find the line of intersection of the plane, M, in Exercise 31 with the plane, Q, in Exercise 34. (Draw an auxiliary plane parallel to the front plane, cutting the planes M and Q. It will cut from each of these planes a line parallel to the front plane. The intersection of the two *lines* thus obtained is *one point* in the required line of intersection.)
36. Take the line a $(-2\frac{1}{2}, 0, -\frac{3}{4})$, b $(-2, -1, 0)$, as the line of intersection of two planes R and S. In any direction, through the front projection of a, draw the front traces of the two planes. Determine the horizontal traces.
37. In Fig. 13 the plan * and elevations of the roofs of a stable

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* *Plan* is another name for the horizontal projection, and *elevation* for the front or side projections.

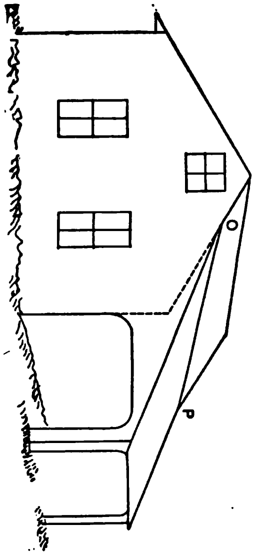


FIG. 14.

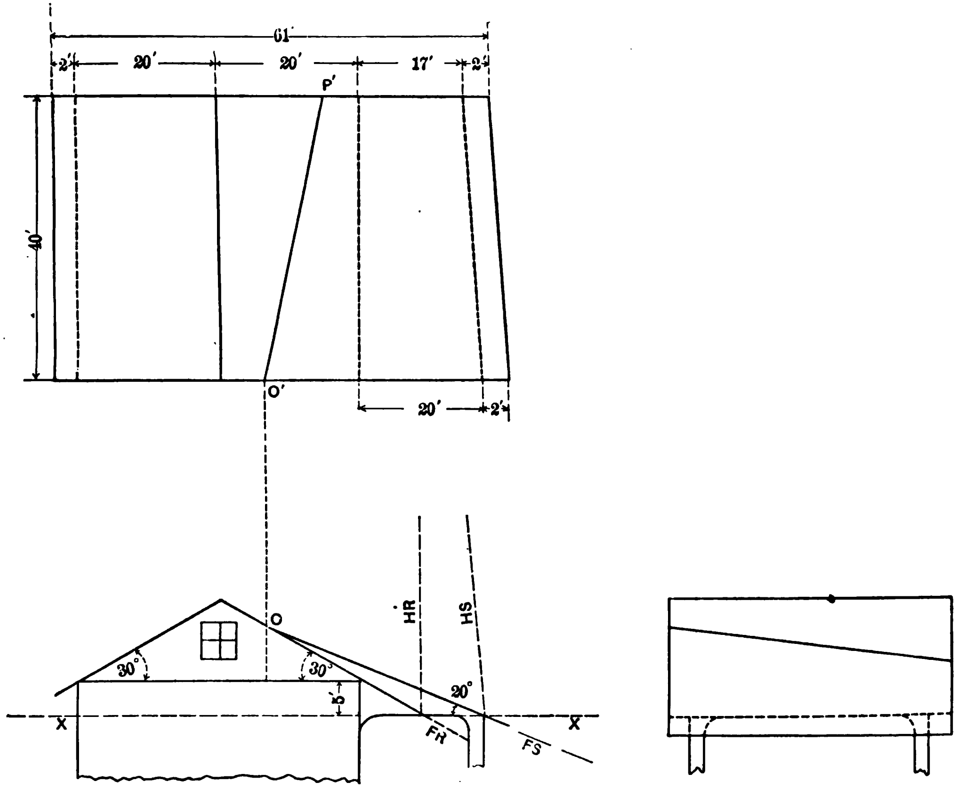


FIG. 13.

and an adjoining shed are shown. Complete the drawings by showing the projections of the line where the two roofs intersect. The dimensions and slopes necessary for drawing the *traces* of the roof planes are given. Mark the stable and shed roofs respectively *R* and *S*. Take scale 1″ = 1′.

Suggestion.—In the figure the horizontal plane of projection is taken through the bottom of the shed roof, thus placing the *X* axis as it is shown in the drawing. The front traces of the roof planes, *F R* and *F S*, are, then, coincident with the lines showing the roof slopes in the front projection. The horizontal traces, *H R* and *H S*, are drawn through the intersections of the front traces with the *X* axis, and parallel to the lines in the plan which show the intersections of the roofs with the horizontal plane of projection. The line of intersection of these two planes, *R* and *S*, determines the *slope* of the line O P—the line showing the intersections of the roofs in the plan.

The perspective drawing (Fig. 14) is added merely to make the other drawings plainer.

25. Of the fundamental problems, the first, that of finding the intersection of a plane with another plane, should be now well in mind. We shall now take up the second of these most important problems, that of finding the intersection of a given line with a given plane. It should be plain that **if another plane,**—called usually an auxiliary plane,—**is passed through the given line, the point where the given line intersects the given plane is in the line of intersection of the auxiliary plane with the given plane.** To illustrate, take two sheets of cardboard and a pencil. Place one sheet on a table and hold the pencil in any way to intersect this sheet. Now hold the other sheet so that one of its edges touches the first sheet, while at the same time the pencil lies in its plane. Observe that **the pencil now touches the sheet on the table in the line of intersection of the two sheets.** This same principle is shown also in the pictorial drawing in Fig. 15. Here we want to show the intersection of the line *c d* with the plane *Q*. The line is shown by its projections, c^h d^h and c^f d^f, and the plane by its traces, *H Q* and *F Q*. The auxiliary plane, *A*, marked by its traces, *H A* and *F A*, is passed through the line *c d*. The planes

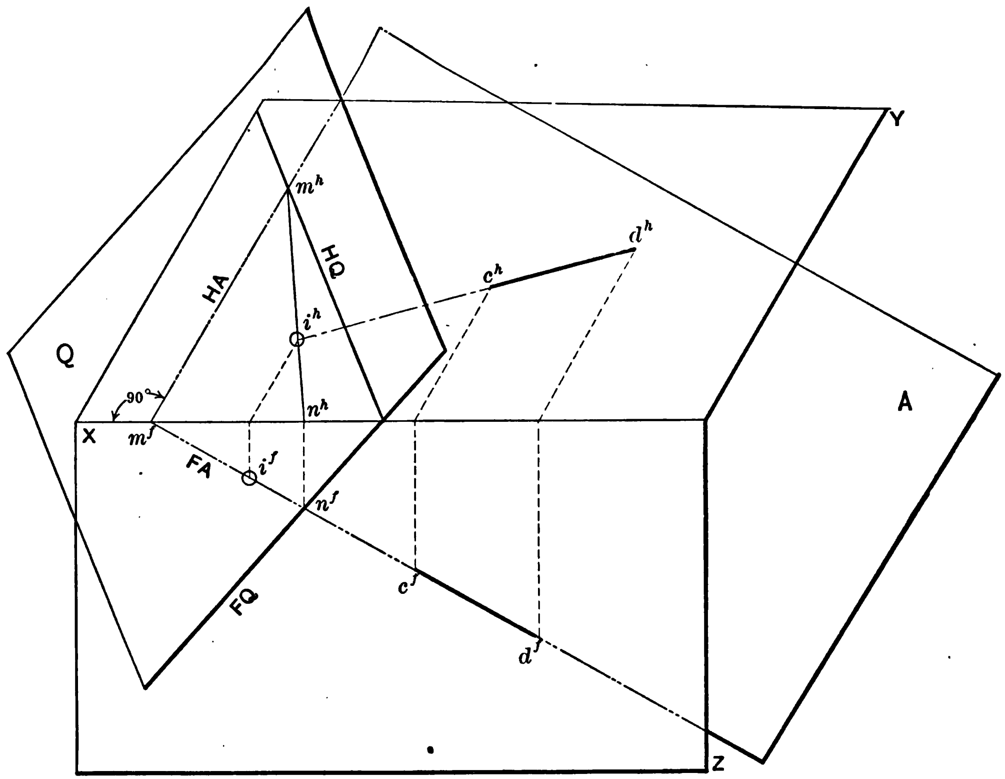


Fig. 15.

Q and *A* intersect in the line represented by the projections, $m^h\, n^h$ and $m^f\, n^f$. **Remember that the point where the line *c d* intersects the plane *Q* must be in the line *m n*.** This required point is also, of course, in the line *c d*. We know, then, that the point we are seeking is in both of the lines *m n* and *c d*, and therefore at the point, *i*, where they intersect.

The same problem is shown in orthographic projection in Fig. 16. The line *c d* is shown by its projections and the plane *Q* by ts traces. The auxiliary plane, *A*, is passed through the line *c d*. The solution is general, so that any other plane including the line *c d* could be taken for the auxiliary plane. Fewer lines are needed for the drawing, however, if the auxiliary plane is drawn perpendicular to one of the planes of projection. In Fig. 15 it is perpendicular to the front plane, so that the horizontal trace, *H A*, is perpendicular to the *X* axis. The planes, *Q* and *A*, intersect in the line *m n*; and the point where the line *c d* intersects the plane *Q* is at the intersection of *m n* and *c d*,—at the point *i*, shown by its horizontal and front projections i^h and i^f. In this particular case, the projection i^h is determined by the intersection of the horizontal projections of *m n* and *c d*. The front projection, i^f, is found by drawing a projecting line through i^h to intersect the front projections of *m n* and *c d*, which are coincident.

26. PROBLEM 7. *To find the point in which a given line intersects a given plane.*

Method. Pass *any* plane through the given line (usually a plane perpendicular to the front plane). The **line of intersection** of this auxiliary plane with the given plane, intersects the **given line** at the point that is required.

EXERCISES

38. Find the point where the line *j* $(-2\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4})$, *k* $(-1\frac{1}{2}, -\frac{3}{4}, -1)$ intersects the plane *U* (0, 135°, 240°).
39. Determine where the line *j k* given in the preceding exercise intersects the plane *W* (0, 45°, 210°).

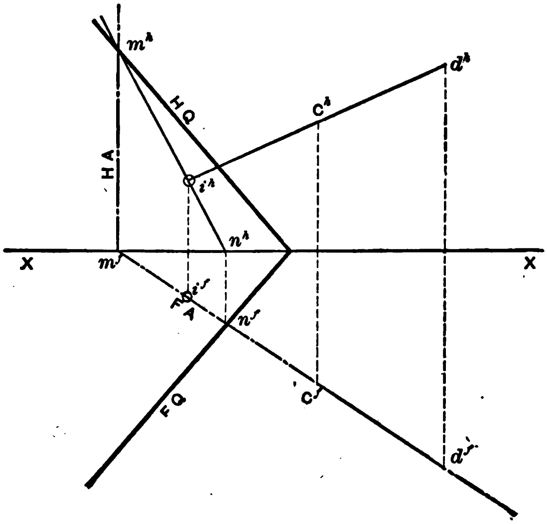
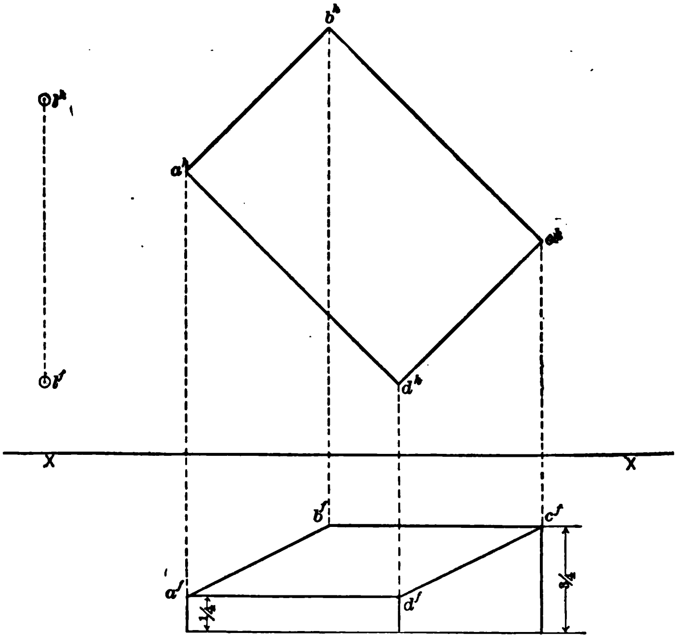


Fig. 16.

40. Draw the plane P ($-3\frac{1}{2}$, 120°, 225°). Find where the line a ($-1\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{1}{4}$), b (0, -2, $-1\frac{1}{2}$) intersects it.

41. Locate the point where the line c ($-1\frac{3}{4}$, $-\frac{1}{4}$, 0), d (-1, -1, $-\frac{1}{2}$) passes through the plane Q of which both the horizontal and front traces are parallel to the X axis, and are respectively $\frac{3}{4}$ inch behind and $1\frac{1}{4}$ inches below this axis.

42. The top of the desk shown in the accompanying figure is located by the points a (-10, -8, -4), b (-6, -12, -2), c (0, -6, -2), and d (-4, -2, -4). A light placed at l (-14, -10, $+2$) has its rays reflected so that the most intense light is inclined 60° from the *vertical*. Show the curve of intersection of the rays of maximum intensity with the plane of the desk.



43. The plane M ($-1\frac{1}{4}$, 30°, 315°) represents a mirror. Find the point where a ray of light passing through the points

r (-2, $-1\frac{1}{4}$, -1) and s ($-1\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{1}{2}$) is reflected from the surface of the mirror.

44. A steam-pipe in a building passes through a sloping floor. The axis of the pipe is located by the points v ($-2\frac{1}{4}$, $-\frac{1}{4}$, $-\frac{1}{2}$) and w ($-\frac{1}{2}$, $-\frac{3}{4}$, $-1\frac{1}{2}$). The plane of the floor may be represented by the plane F ($-\frac{3}{4}$, 120°, 210°). Find the intersection of the axis of the pipe with the floor.

27. **Lines that are parallel to a plane appear in their true length in their projections on that plane.** In Fig. 17 the horizontal and front projections of a hip roof are shown. Lines that are parallel either to the horizontal plane or to the front plane, such as the lines $c\,d$ or $c\,e$ in the figure, are here drawn in their true length. A line that is oblique, however, to both planes of projection, as for example the line $b\,f$, is not shown in its true length in the drawing; but in this case a third view may be made to show its true dimensions. This is illustrated in the right-hand drawing in the figure. The roof is here projected on a vertical plane, V, drawn through the line $b\,f$, and then this vertical plane is revolved into the plane of the drawing. This projection is constructed by drawing the base line, $V'\,V'$, parallel to $V\,V$. At g' on the projecting line $b^h\,b'$ produced, the line $g'\,b'$ is laid off perpendicular to $V'\,V'$. Its length is equal to the "altitude" of the point b, or to the distance $b^f\,g^f$ in the front view. The line $b'\,f'$ shows then the true length of $b\,f$. The completed drawing shows also the true length of the line $a\,c$, as it is parallel to the vertical plane, V.

28. In practice these constructions are simplified by passing the auxiliary vertical plane *through* $b^h\,f^h$. The line $V\,V$ is the trace of this plane in the plane of the base of the roof. If this plane is revolved, together with the line $b\,f$, about $V\,V$ to coincide with the plane of the base, the line $b\,f$ will then lie in a plane where true dimensions can be measured.

The revolution of one line about another line as an axis is accomplished usually by the revolution of two points in the line. A point revolved thus about a line as an axis describes a circle with

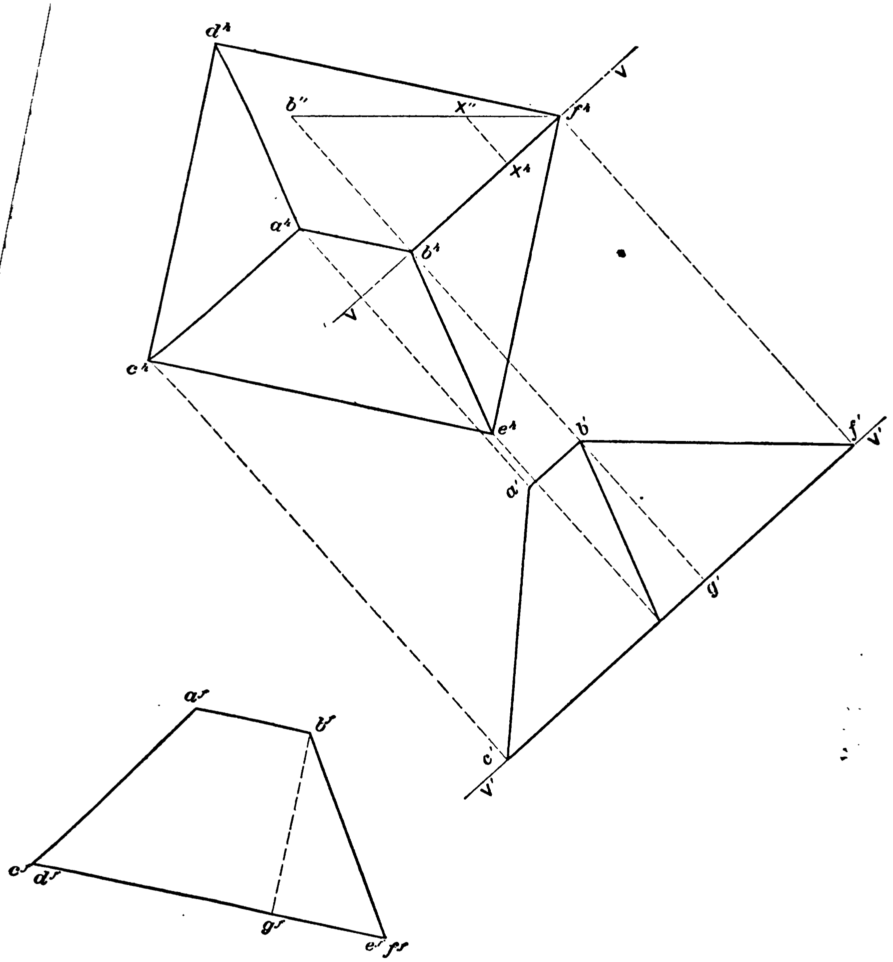


FIG. 17.

a radius equal to the actual perpendicular distance from the point to the axis. In this case the points b and f must be revolved about the axis $V\ V$. The axis is here in the plane of the base, so that the perpendicular distances of the points from it can be measured in the front view. For the point b this distance is $g^f\ b^f$, which is laid off on the line through b^h, perpendicular to $V\ V$. The revolved position of b is at b''. The point f is in the axis, so that in revolution it is stationary. The line $b''\ f^h$ is then the position of $b\ f$ when revolved into a plane of the drawing and shows its true length.

29. If it is required *to lay off a given distance* on a line not shown in its true length in a drawing, we must first revolve the line about an axis to bring it into a plane where its true length is shown and then measure the distance. For example, if we wish to measure a distance, $b\ x$, along the line $b\ f$ (Fig. 17) from the point b, we lay off the given length on the revolved position $b''\ f^h$, locating the point x'', and then reversing the preceding process, we revolve x back into the line $b\ f$. The horizontal projection of $b\ x$ is then $b^h\ x^h$.

30. Usually when the true lengths of lines are to be found, it is most convenient to revolve them into either the horizontal or front planes, rather than into an auxiliary plane which is parallel to one of the planes of projection. Of course any line which lies in one of the planes of projection is shown there in its true length. This method of solution is illustrated in Fig. 18*a*, where a line $m\ n$ is shown as it is located in space, and also by its projections $m^h\ n^h$ and $m^f\ n^f$. The point m in this line lies in the horizontal plane, so that this point is marked only by its projection m^h. This figure shows the line $m\ n$ also when it is revolved into the planes of projection. Let us consider first how it is revolved into the horizontal plane. The point m being already in this plane, in revolution remains stationary. Only the point n therefore must be revolved to show the revolved position of the line. This point moves in the arc of a circle about its horizontal projection m^h as a center, and with a radius equal to the distance the point n is below the horizontal plane. Its revolved position is at n'. In a drawing, however, this revolution is shown by constructing at n^h

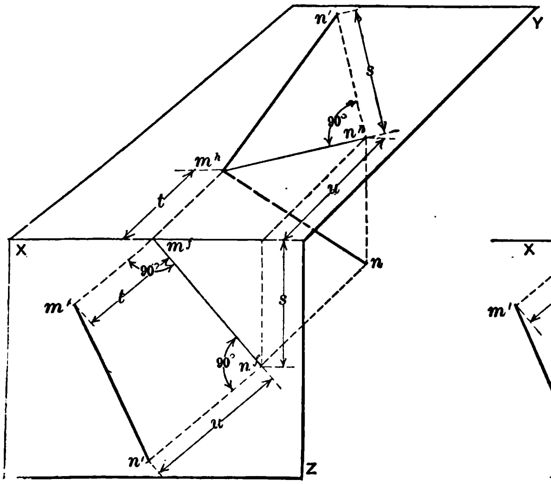


FIG. 18a.

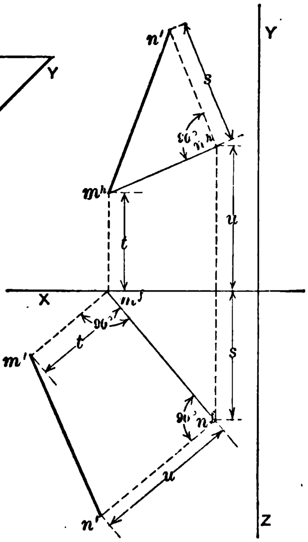


FIG. 18b.

a line perpendicular to the axis, $m^h\ n^h$, and laying off on the perpendicular the distance the point n is below the horizontal plane. By the dimension lines in the figure, this distance is marked s. The line $m^h\ n'$ is thus obtained. It is a line in the horizontal plane and is, therefore, shown in its true length. In the same way the true length of the line was found by revolving into the front plane. The points m and n revolve now respectively to m' and n' in the front plane. Dimension lines t and u show the distances that, in this case, are equal. The true length is shown by $m'\ n'$. If the line $m\ n$ is revolved into the side plane, the same result is obtained.

Figure 18*a* shows all this very plainly as in a picture. This method, however, is the same for problems in orthographic projection. In this way, the same line $m\ n$ is represented in Fig. 18*b* by its horizontal and front projections, and its true length is shown now very accurately by $m^h\ n'$ and $m'\ n'$. The student should measure these lengths to satisfy himself that they are equal.

A case occurs sometimes where the horizontal and front planes have not been advantageously located, so that lines pass through and continue beyond the planes of projection. Then the projections of parts of the line fall on opposite sides of the X axis. When such a line is revolved into the plane which it passes through, distances must be laid off on opposite sides of the axis of revolution.

In Fig. 19 a line $a\ c$ is shown intersecting and passing through the horizontal plane at b. If this line is revolved into the horizontal plane about $a^h\ c^h$ as an axis, the point b remains stationary, and a and c revolve in *opposite directions*. The distance that a^f is below the X axis is measured on one side of $a^h\ c^h$, and the distance that c^f is above is measured on the other side. Then $a'\ c'$ is the true length of the line. If, however, the same line is revolved into the front plane, this difficulty is not met, as $a''\ c''$ shows now the true length.

31. Problem 8. *To find the true length of a line given by its projections; or to find the distance between two points.*

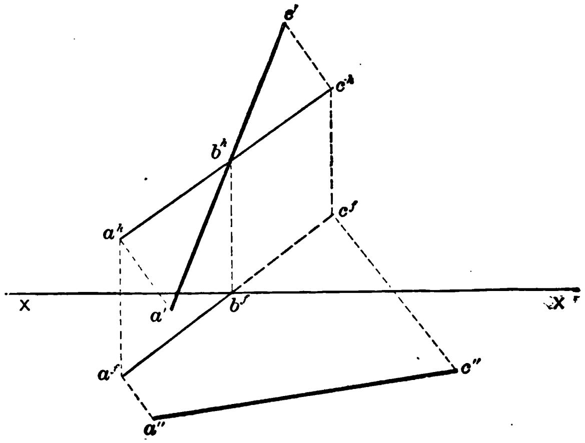


FIG. 19.

Method. Pass an auxiliary plane through the line perpendicular to a plane in which lines are shown in their true lengths on the drawing (usually a plane of projection). With the line of intersection of the two planes as an axis, revolve the line into the second plane, where it is shown in its true length.

EXERCISES

45. Find the distance between the points a $(-5, -1, -\frac{1}{2})$ and b $(-1, -3, -\frac{1}{2})$.
46. Find the true distance between the points c $(-6, -1, -2)$ and d $(-\frac{1}{2}, -2, -\frac{1}{2})$.
47. Locate, in a plane parallel to the front plane, two points that are two inches apart and are not equal distances below the horizontal plane.
48. A line passes through the points e $(-5, 0, -1\frac{1}{2})$ and f $(-1, -2\frac{1}{2}, 0)$. Draw the side projection of this line. Find the length of this line included between its intersections with the horizontal and front planes; and with the front and side planes.
49. Two stations s_1 and s_2 are to be connected by a telegraph line. The locations are s_1 $(-3, -1, -\frac{1}{2})$ and s_2 $(+\frac{1}{2}, -\frac{1}{2}, -2)$. Find the length of the shortest line that will connect the two stations.
50. Find the length of the shortest belt to connect two pulleys. Both are two inches in diameter and are in the same plane. Their centers are at b $(-3\frac{1}{2}, -1, -\frac{1}{2})$ and c $(-1\frac{1}{2}, -\frac{1}{4}, -1)$.
51. A line is $4\frac{3}{4}$ inches long. The projections of its ends are located by the points a $(-5, -1, -\frac{1}{2})$ and b $(-1, -3, ?)$. Determine the projection that is not given.
52. A line lies in the plane M $(-3, 30°, 330°)$. One projection of the line is given by the points g $(-1\frac{1}{2}, -\frac{1}{4}, ?)$ and h $(-\frac{1}{2}, -1, ?)$. On this line lay off $1\frac{1}{2}$ inches from its intersection with the front plane.

32. Lines Perpendicular to a Plane. Two planes P and H are shown in Fig. 20. A line $a\,c$ is perpendicular to the plane P. From the point a the projecting line $a\,a^h$ is drawn perpendicular to the plane H. The plane determined by the lines $a\,c$ and $a\,a^h$ is perpendicular to the planes P and H. It is, therefore, perpendicular to the trace $H\,P$ which is the intersection of the planes. Then $a^h\,c^h$, which is a line lying in this plane, is perpendicular to $H\,P$. Consider now the plane H as a plane of projection; then $a^h\,c^h$ is the projection of the line $a\,c$, while $H\,P$ is the trace of the plane P. We have shown, then, that when a line $a\,c$ is perpendicular to a plane P, its horizontal projection is perpendicular to the horizontal trace. The same relation holds, of course, for the other projections of the line and the corresponding traces of the plane.

If, then, a straight line is to be drawn perpendicular to a plane, it is only necessary to draw its projections perpendicular to the corresponding traces of the plane. And to draw a plane perpendicular to a line, draw the traces perpendicular to the projections of the line.

33. PROBLEM 9. *To find the distance from a given point to a given plane.*

Method. Draw through the given point a line *perpendicular* to the plane. Find the point where this line intersects the given plane. The required distance is the true length of the line joining this last point with the given point.

EXERCISES

53. A plane Q $(-2, 45°, 330°)$ is perpendicular to a line drawn through the point j $(-2, -1\frac{1}{2}, -1)$. Draw the projections of the line; and find the distance from j to Q.
54. In Ex. 42 find the distance from the point l to the plane of the top of the desk.
55. Find the distance from the point w in the axis of the steam-pipe given in Ex. 44 to the plane of the floor.
56. A hillside is represented by the plane S $(0, 120°, 210°)$. A

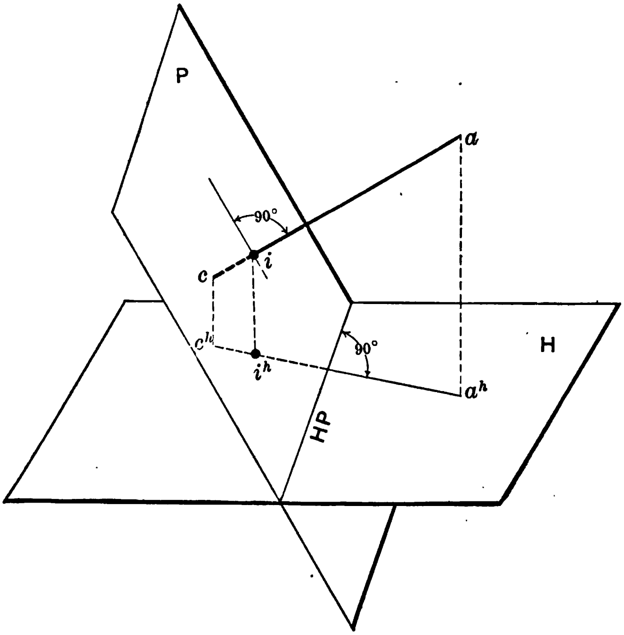


Fig. 20.

pole is planted on it perpendicular to its plane. The bottom of the pole passes through the point t $(-1\frac{1}{4}, -1, -1\frac{1}{4})$. Measuring by the same scale as for the coördinates of t, locate a point that is 1 inch above the ground. What is the length of the part of the pole in the ground?

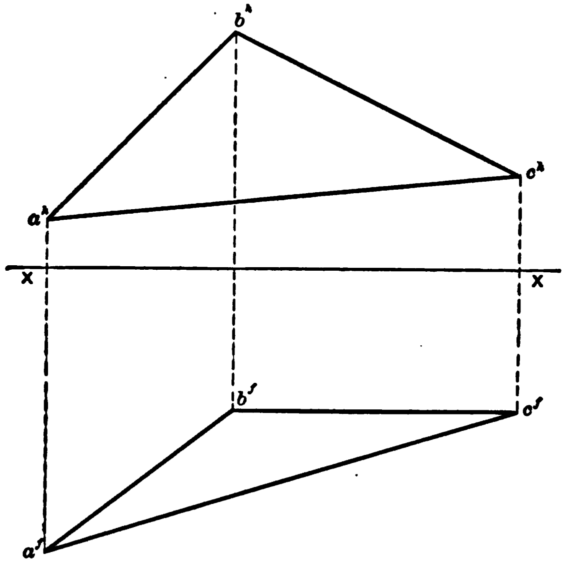
57. A point i $(-1\frac{3}{4}, -\frac{1}{2}, -\frac{1}{2})$ is above the roof represented by the plane R (0, 150°, 225°). A shaft for transmitting power to another building passes through this point perpendicular to the roof. Locate points on the shaft 1 inch above the roof and 2 inches below.
58. Draw the projections of a cube and find the distance from any corner to an oblique plane passing through it. The plane should not be parallel to an edge.
59. Find the distance between two parallel planes, of which all the traces are oblique to the X axis.

34. PROBLEM 10. *To project a given line upon a given plane.* *Method.* Select any two points in the line and through each, draw the projections of a line perpendicular to the plane. The line joining the points where these perpendicular lines intersect the given plane is the projection of the line upon that plane.

EXERCISES

60. Project the line m $(-1\frac{3}{4}, -1, -1)$, n $(-\frac{3}{4}, -\frac{3}{4}, -1)$ upon the plane P (−1, 135°, 210°).
61. The major and minor axes of an elliptical cam-wheel are determined respectively by the lines d $(-1\frac{3}{4}, -1\frac{1}{4}, -\frac{1}{2})$, e $(-\frac{1}{2}, -1\frac{1}{4}, -\frac{1}{2})$; and f $(-1\frac{1}{8}, -1\frac{3}{4}, -\frac{1}{2})$, g $(-1\frac{1}{8}, -\frac{3}{4}, -\frac{1}{2})$. Draw the projection of this cam-wheel on the plane T (−5, 30°, 315°) by projecting a number of diameters of the ellipse.
62. The vertices of a triangle shown in the figure are at the points: a $(-2\frac{1}{2}, -\frac{1}{4}, -1\frac{1}{2})$, b $(-1\frac{1}{2}, -1\frac{1}{4}, -\frac{3}{4})$, c

(0, $-\frac{1}{2}$, $-\frac{3}{4}$). Project this triangle upon the plane ***R*** ($+1\frac{1}{2}$, 135°, 240°).



35. If one line is given by its projections, and a plane is to be drawn perpendicular to it, it follows from the explanation in Art. 32 that the traces of the required plane must be drawn perpendicular to the corresponding projections of the given line. In Figs. 21*a* and 21*b* a line *a b* is shown by its two projections, and it is required to draw through a given point *o* a plane perpendicular to the line *a b*. Through the point *o* draw a line *m o* parallel to the horizontal plane which shall lie in the required plane. A line which is parallel to the horizontal plane is commonly called a "horizontal," and is a line with its front projection parallel to the X axis. And if this "horizontal" shall lie in the required plane, its horizontal projection must be parallel to the horizontal trace of the plane to be drawn. In other words, in this case, the conditions to be satisfied are: (1) that the front projection m^f o^f of the "horizontal" must be drawn parallel to the X axis; and (2) that the horizontal projection m^h o^h must be drawn perpendicular

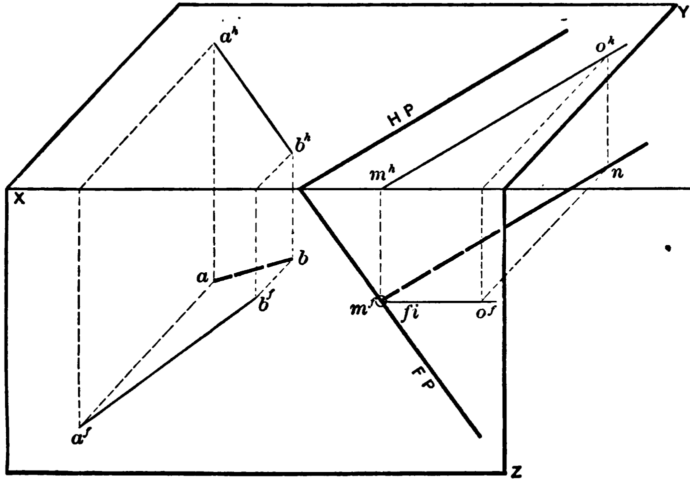
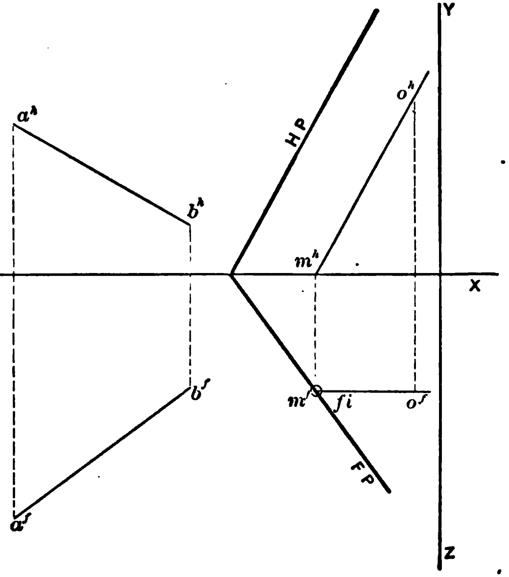


Fig. 21*a*.



Fi 21*b*.

to the horizontal projection of the given line $a\ b$—the same as saying it must be parallel to the horizontal trace of the required plane.

Having thus determined a line $m\ o$ lying in the required plane, the front trace, $F\ P$, is drawn through the intersection ($f\ i$) of the line $m\ o$ with the front plane, and perpendicular to $a^f\ b^f$. The horizontal trace, $H\ P$, is drawn through the intersection of $F\ P$ with the X axis and perpendicular to $a^h\ b^h$. A plane, P, perpendicular to the line $a\ b$ and passing through a given point o, is thus determined.

A line parallel to the front plane and intersecting the horizontal plane (determining the horizontal trace first) might, of course, be used in the place of the "horizontal," to obtain the same result.

36. PROBLEM 11. *To pass a plane through a given point perpendicular to a given line.*

Method. Draw through the given point a line which will lie in the required plane (a "horizontal" line is usually most easily used). Through the intersection of this line with one of the planes of projection draw one trace of the required plane perpendicular to the corresponding projection of the given line. The other trace is drawn perpendicular to the corresponding projection of the line, and through the intersection of the first trace with the X axis.

EXERCISES

63. Draw through the point e $(-1, -1, -1\frac{1}{4})$ a plane, U, which shall be perpendicular to the line g $(-3, -1\frac{1}{4}, -1)$, h $(-2, -\frac{1}{4}, -\frac{1}{2})$.

64. The points b $(+3, 0, -1\frac{1}{2})$ and c $(+6, -5, -1\frac{1}{2})$ are in the eaves of a roof. The point g $(+1\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{2})$ is in the ridge. Draw through the point w $(0, -2\frac{1}{2}, -1\frac{1}{2})$ the *projections* of an arrow that will show the direction of a force due to wind pressure on the roof. Make the tip of the arrow *touch* the surface of the roof. (Pressures of fluids are perpendicular to the planes on which they are exerted.)

65. Draw a plane parallel to the plane P (0, 120°, 225°) at a distance of two inches from it on the right-hand side.

37. In a complicated drawing it is often necessary to find the true size of the angle between lines in a plane surface which is not parallel to any plane showing true dimensions. The usual process is to imagine the plane of the surface extended to intersect the planes of projection, thus determining the horizontal and front traces of the plane in which the surface lies. About one of these traces as an axis, the plane is then revolved till it coincides with the horizontal or front planes.

The simplest case of finding the true size of a plane surface is illustrated at the left-hand side of the roof shown in Fig. 22. The side $a\ b\ c$ is not shown in its true size, neither are its angles. If, however, the side is revolved about its intersection with the plane of the base of the roof, $b^h\ c^h$, into the plane of the base, its true size will be seen. Then the lines $a\ b$ and $a\ c$ will be seen in their true lengths and with **the true size of the angle between them.** In this revolution about the axis, $a^h\ b^h$, the path traced by the point a is shown in its front projection by the dotted lines of the arc of a circle through a^f, with its center at $b^f\ c^f$. The point a' shows its revolved position in the plane of the base, and $a'\ b^h\ c^h$ shows true dimensions for any part of the outline of the surface.

38. In the preceding case, the problem was simplified because the surface to be determined was perpendicular to the front plane of projection, so that the radius for the revolution of the corner a was shown in its true length in its front projection. At the right-hand side of Fig. 22, where the side of the roof $d\ e\ f$ is represented, a more difficult problem is presented. Here when the revolution is made about the line $e^h\ f^h$ (the intersection of the side with the plane of the base) as an axis, the point d will move in the arc of a circle with a radius equal to the perpendicular distance from d to the axis. Obviously the line $d g$ is this radius. The perspective view in Fig. 23 shows this radius, and by the arrow on $d d'$ the direction of revolution. The *true* length* is found easily from its

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* The true length is measured by the hypothenuse of the right triangle $d\ d^h\ g^h$ in the horizontal projection. In this triangle $d\ d^h$ is laid off equal to the distance from d^f to the line of the base of the roof $b^f\ c^f\ e^f\ f^f$.

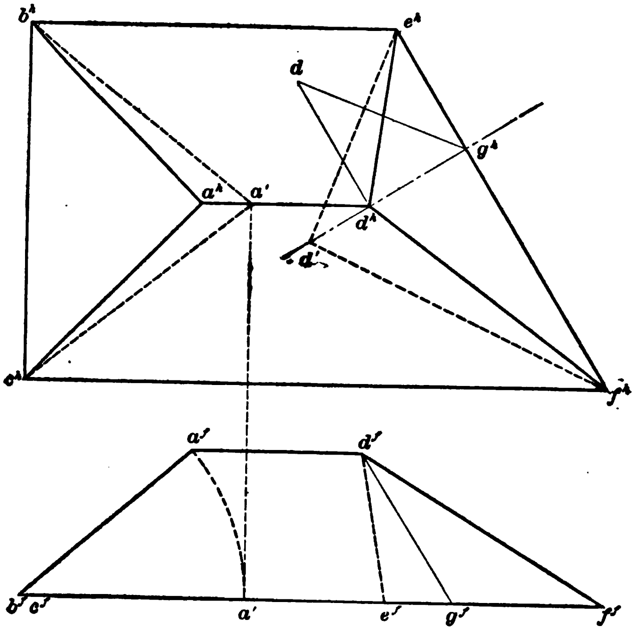

Fig. 22.

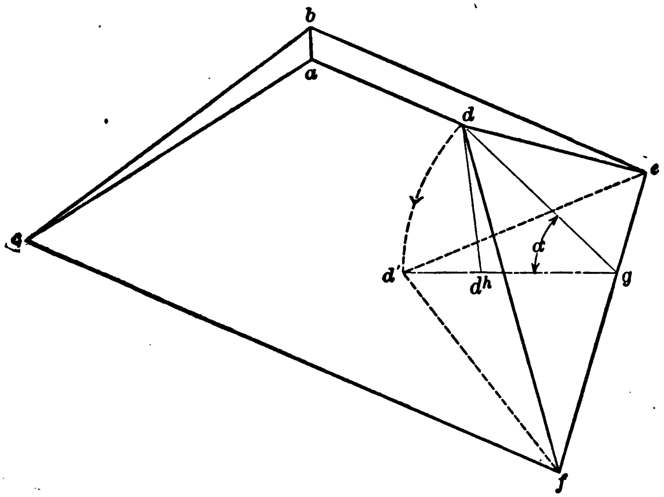

Fig. 23.

two projections. The true length, $d' g^h$, of ~~this radius when~~ laid off on $d^h g^h$ produced, shows the revolved position of the corner, d, at d'. The true size of the given surface is shown by the triangle $d' e^h f^h$, from which true angles can be measured.

The perspective view of this roof represented in Fig. 23 shows more plainly the right triangle called $d\, d^h g^h$ in the preceding construction, together with the arc described by the point d in revolving to d' about $e^h f^h$ as an axis.

The right triangle $d\, d^h g$ (Fig. 23) shows also the true angle the side $d\, e\, f$ makes with the plane of the base. This angle is marked α in the figure.

39. Fig. 24 shows the simplest statement of the problem to find **the true angle between two intersecting lines** $m\, n$ and $o\, p$. None of the angles between the projections of these lines show the true angle between the lines; and they must be revolved into a plane where true dimensions are shown. Before, however, the lines can be revolved into one of the planes of projection, the traces of the plane in which they lie must be determined. The intersections of the lines with the horizontal and front planes are marked respectively $h\, i$ and $f\, i$. The lines joining these points that are corresponding are the traces of the required plane. The lines must then be revolved about one of the traces into the corresponding plane of projection; that is, we must revolve about the horizontal trace into the horizontal plane, about the front trace into the front plane, etc. In the figure the lines are shown revolved into the horizontal plane about the horizontal trace, $H\, P$, as an axis. The true length of the radius for revolving the point of intersection, i, of the two lines is found by constructing the right triangle * shown at the right of the drawing. The hypothenuse of this triangle is equal to the true distance from i to $H\, P$. The revolved position of i is shown at i' by laying off a distance equal to the length of the hypothenuse at right angles to $H\, P$. The ends of the lines $m\, n$ and $o\, p$ are

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* In practice, usually, this right triangle can be placed on the drawing so that only one leg must be transferred. For example, instead of constructing the right triangle as shown, the distance marked a might have been laid off with dividers on the X axis, at right angles to the distance marked b and the hypothenuse could be measured in one operation.

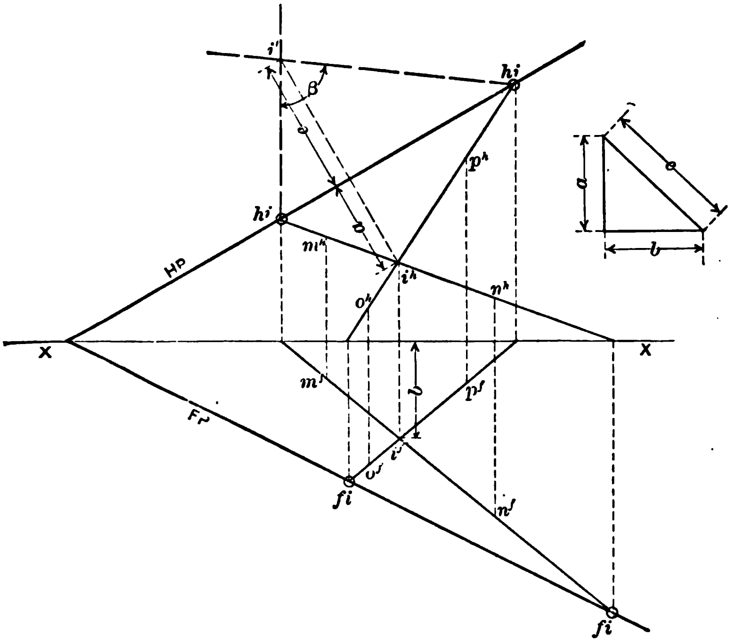


FIG. 24.

in *H P;* and in revolving the lines, these points remain stationary. The lines shown by heavy dashes in the figure represent the lines mn and op when they are revolved into the horizontal plane. The angle β is the true size of the acute angle between the two lines. Observe that for the construction only one trace is needed, the other might be omitted.

40. PROBLEM 12. *To find the true size of the angle between two intersecting lines.*

Method. Determine one of the traces of the plane in which the given lines lie. With this trace as an axis, revolve the two lines into the plane of projection in which the axis lies. The angle between the lines in their revolved position is the true size of the required angle.

EXERCISES

66. Determine the plane of the lines $a\,b$ and $b\,c$ given by the coördinates: $a\ (-2, -\frac{1}{2}, -\frac{3}{4})$, $b\ (-1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{4})$, and $c\ (-1\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$. With the horizontal trace of this plane as an axis, revolve these lines into the horizontal plane.

67. The line $a\ (-2, -\frac{1}{2}, -\frac{3}{4})$, $b\ (-1, -1\frac{1}{2}, -\frac{1}{4})$ intersects the line $c\ (-2, -1\frac{1}{4}, 0)$, $d\ (-\frac{1}{2}, -\frac{1}{2}, -1\frac{1}{2})$ at $e\ (-1\frac{1}{2}, -1, -\frac{1}{2})$. Find the true size of the obtuse angle between these two lines.

68. A boat is towed along a canal by two mules, one walking on each bank. In projection drawing, the boat is shown at $b\ (-2, -\frac{1}{2}, -1)$. The tow-paths are not at the same level, and the positions of the mules are shown at $m_1\ (-1, -1, -\frac{1}{4})$ and $m_2\ (-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2})$. Find the true size of the angle between the tow-ropes.

69. If in the preceding exercise the forces exerted by the two mules are the same, show the course the boat will take.

    Suggestion. If the forces in the two tow-ropes are equal, the course of the boat is shown by the bisector of the angle between them.

70. The vertices of a triangle are at the points a $(-3, -\frac{1}{4}, -1\frac{3}{4})$, b $(-2, -1\frac{1}{4}, -\frac{3}{4})$, and c $(-\frac{1}{2}, -\frac{1}{2}, -\frac{3}{4})$. Draw the projections of the bisector of the angle between the sides $a\,b$ and $b\,c$ on the horizontal, front, and side planes.

71. The mast of a derrick is shown by the line c $(-2, -1\frac{1}{2}, -\frac{3}{4})$, d $(-2, -1\frac{1}{2}, -2\frac{1}{4})$. Guy-ropes attached to the top of the mast at c are fastened to the ground at a $(-4\frac{3}{4}, -2, -2)$ and b $(-4, -\frac{1}{4}, -2\frac{1}{2})$.

(*a*) Find the angle between the guy-ropes $a\,c$ and $b\,c$.

(*b*) Find the angle between the rope $b\,c$ and the mast $c\,d$.

Check this last result by revolving $b\,c$ into the front plane.

Suggestion. Observe that the $h\,i$ for $c\,d$ is at $c^h\,d^h$ in the drawing.

41. The angle which a line makes with its projection on any plane measures the true angle between the line and the plane. By drawing a line perpendicular to the plane from a point in the given line a right-angled triangle is formed, in which the perpendicular is one leg, the projection of the line is another leg, and the line itself is the hypothenuse. The required angle is that between the hypothenuse and the leg lying in the plane; but the other acute angle of this triangle, the complement of the angle between the line and the plane, is more easily found by the method of Art. 40.

42. PROBLEM 13. *To find the angle between a given line and a given plane.*

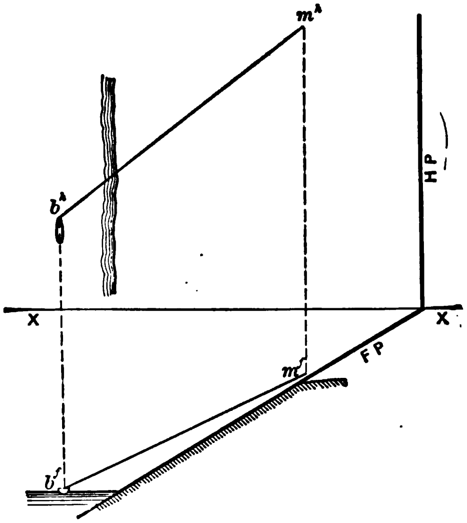
Method. From any point in the given line draw a line perpendicular to the plane. Find the angle between these two lines. The complement of this angle is the required angle between the line and the plane.

EXERCISES

72. Find the angle the line a $(-1\frac{3}{4}, -1, -1)$, b $(-\frac{3}{4}, -\frac{1}{2}, -1\frac{3}{4})$ makes with the plane P $(-1, 135°, 210°)$.

73. Find the angle the line c $(-2, -\frac{3}{4}, -1)$, d $(-1, -\frac{1}{4}, -\frac{3}{4})$, makes with the plane M $(-3, 135°, 240°)$. (Observe that the angle most conveniently found in this case is the supplement of the angle between the perpendicular and the given line as shown in the right triangle mentioned in Art. 41.)

74. The slope of the bank of a canal makes an angle of 30° with the water-level, so that the plane of the bank is represented by P $(0, 90°, 210°)$ as shown in the figure. A mule on the bank draws a boat in the canal. The traces on the mule are attached to a rope at a point m $(-1\frac{1}{4}, -3, -\frac{5}{8})$. The other end of the rope is fastened to a boat at b $(-4, -1, -1\frac{3}{4})$. Assuming there is no sag in the rope (that the points m and b in the figure are in a straight line), find the true angle between the rope and the bank of the canal.



75. In constructing shadows on drawings we take rays of light that are reflected downward from the left-hand side, and are shown by lines whose horizontal and front projections

make respectively angles of 45° and 315° with the X axis.

(*a*) Find the angle between these rays and a roof R (0, 120°, 210°).

(*b*) Find the angle between these rays and the horizontal and front planes of projection.

76. A ray of light passes through the point p $(-1\frac{3}{4}, -\frac{1}{2}, -1)$ and is reflected from a point r $(-\frac{1}{2}, -1, -\frac{1}{2})$ in a plane mirror represented by M (0, 90°, 225°). Find the angle of reflection,* and the distance of the point p from the mirror.

77. Find the angle between a line with both projections parallel to the X axis and the plane Q (0, 135°, 225°).

Suggestion. Since the plane of the given line and the perpendicular must include a line parallel to the X axis, its traces are parallel to the axis.

43. In Fig. 25 two planes, $L\,M\,N$ and $L\,N\,O$, are shown in a pictorial drawing, and we wish to determine the angle between them. If from any point p we draw the lines $p\,q$ and $p\,r$ perpendicular respectively to the planes $L\,M\,N$ and $L\,N\,O$, these perpendicular lines will determine a plane which is perpendicular to both of the given planes, and intersects them in the lines $q\,e$ and $e\,r$. The angle $q\,e\,r$ is therefore the angle between the two given planes. This angle is not usually determined directly. The method ordinarily used is to find the true angle $q\,p\,r$ between the intersecting lines $p\,q$ and $p\,r$ (Art. 40). This angle is the supplement of the angle $q\,e\,r$. Of course either of the angles $q\,e\,r$ (ϕ) or $q\,p\,r$ $(180° - \phi)$ is the angle between the two planes.

The above method is practically the same as finding the angle between the lines cut from the two given planes by an auxiliary plane passed through them, perpendicular to their line of intersection.

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* The angle of reflection is *equal* to the angle between the ray and the mirror before reflection, which is called the angle of incidence.

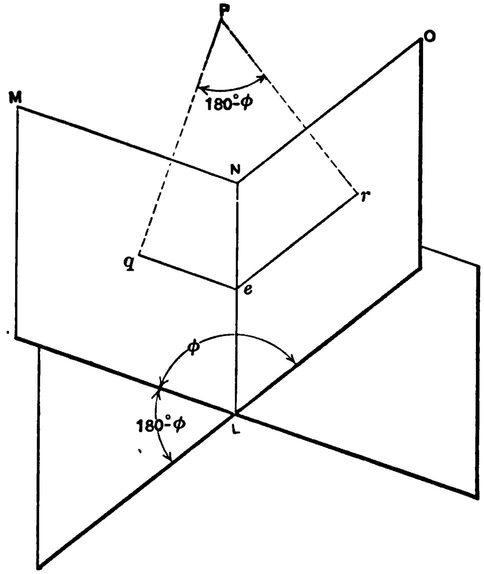


Fig. 25.

44. PROBLEM 14. *To find the angle between two oblique planes.* *Method.* From any point in space draw a line perpendicular to each of the given planes. The true angle between these two lines is the angle between the planes.

EXERCISES

78. Find the angle between the planes S (-3, 45°, 315°) and T (0, 120°, 210°).

79. The corners of a hip-roof are given by the points a $(-2\frac{1}{2}, -\frac{1}{2}, -1)$, b $(-\frac{1}{2}, -\frac{1}{2}, -1)$, c $(-\frac{1}{2}, -2, -1)$, and d $(-2\frac{1}{2}, -2, -1)$. The ridge is the line e $(-2, -1\frac{1}{4}, -\frac{1}{2})$, g $(-1, -1\frac{1}{4}, -\frac{1}{2})$. (1) Find the angle between the sides of the roof $c\,d\,e\,g$ and $a\,b\,g\,e$. (Use the side traces of the planes) (2) Find the angle between the sides of the roof $a\,b\,g\,e$ and $g\,b\,c$.

80. Find the angle between the plane R (-1, 90°, 225°) and a plane parallel to the X axis with its horizontal and front traces respectively 1 inch behind and $1\frac{1}{2}$ inches below the X axis.

    Suggestion. The intersection of one of the perpendiculars with either the horizontal or the front plane is found easily by using the side projections.

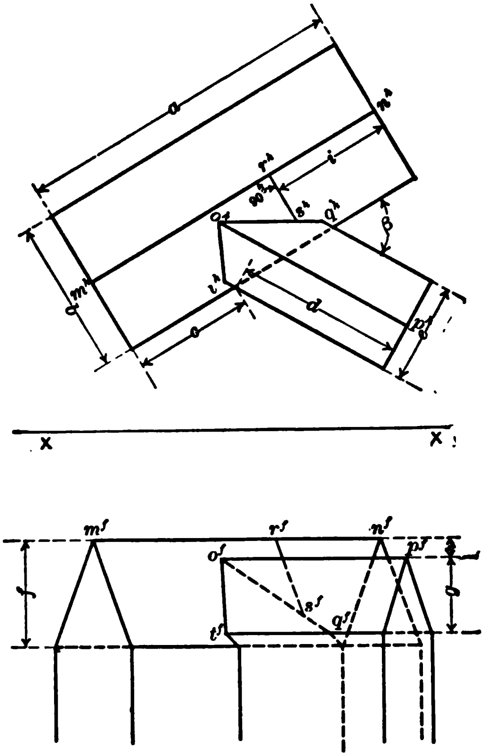
81. Make a drawing similar to the figure on p. 76, taking the following data: Draw $m^h\,n^h$ with a slope of 30° to the X axis. Take $\beta = 60°$, $a = 4$, $b = 2$, $c = 1\frac{1}{2}$, $d = 2$, $e = 1\frac{1}{4}$, $f = 1\frac{3}{4}$, $g = 1$, $h = \frac{1}{2}$. Complete the drawing by determining accurately the lines $o\,t$ and $o\,q$.

    (1) Find the angle between the roof plane $t\,o\,p$ and the one in which the line $r\,s$ lies.

    (2) The line $r\,s$ represents a rafter in the roof. Find the angle of the bevels to be cut at its ends, so that it may be joined to the ridge at r, and to the valley rafter $o\,q$ at s.

82. The corners and ridge of the irregular hip-roof shown in the figure at the top of p. 78 are located by the following

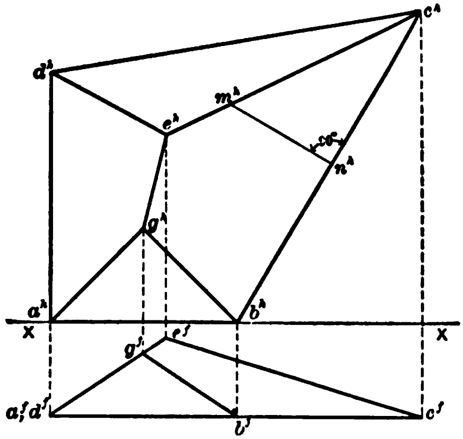
points: *a* (0, 0, −1½), *b* (+3, 0, −1½), *c* (+6, −5, −1½), *d* (0, −4, −1½), *g* (+1½, −1½, −½), *e* (?, −3, ?). Determine the point *e* by finding the line of intersection of the sides meeting in the ridge (*g e*).



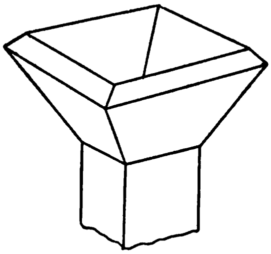
(*a*) Find the angle between the sides of the roof meeting in the ridge.

(*b*) Find the angle of bevel for the top side of the hip-rafter *e c*; that is, find the angle between the planes of the sides *d e c* and *b c e g*.

(*c*) The line *m n* represents a rafter in the side *b c e g*. Find the angle of bevel for its end at *m* to join the hip-rafter *e c*. The front projection of *m n* must be supplied.



83 The figure below shows a square butt-jointed hopper. The edges of the outside top and bottom squares are respectively 5 inches and 2 inches. The outside sloping edge is 3 inches long. Find the angle of bevel for the joints between the sides of the hopper.



84. By the general method used in the preceding exercises, find the angle between the plane P (0, 120°, 210°) and the horizontal plane of projection.

85. The horizontal trace of a plane makes an angle of 45° with the X axis, and the plane itself makes an angle of 60° with the *front* plane. Draw the front trace of the plane.

86. Take the data given for the skew bridge in Ex. 27 and find

(*a*) The angle between the plane of the portal and a horizontal plane.

(*b*) The angle between the plane of the portal and a vertical plane through one of the end posts.

(*c*) The angles between the portal strut *b d* and the end posts *a b* and *c d*.

(*d*) The angle between the portal strut *b d* and the line of intersection of a vertical plane through one of the end posts with a plane through the portal strut perpendicular to the plane of the portal.

Suggestion. A simple way to pass a plane through a given line perpendicular to a given plane is to draw through any point in the given line a line that will be perpendicular to the given plane. The required plane is determined by these two intersecting lines.

(*e*) Measure in degrees the angle found in (*d*) and the angles marked α and β in the figure. By trigonometry the tangent of the angle found in (*d*) is equal to $\sin \alpha \div \tan \beta$. With this formula check the angle found.

45. A very simple method for finding the angle between two sides of an object is explained in Art. 38. It is shown there how the angle a between the side of the roof *d e f* in Fig. 22 and the plane of the base is found immediately from the projections. The angle between the side *d e f* and the *front* plane cannot, however, be found so easily without finding the traces of the planes. The method to be explained in this article applies particularly to finding the angles between a plane given by its traces and the horizontal and front planes. Observe that the problem is the same as that solved in Ex. 84. The problem, however, appears so often for solution that a special case should be made of it.

If an oblique plane is pependicular to either the horizontal or the front plane of projection, one of its traces will be perpendicular to the X axis, and the other trace will show by its slope the angle the plane makes with one of the planes of projection. In Fig. 26*a* a plane, P, is shown perpendicular to the horizontal plane, and

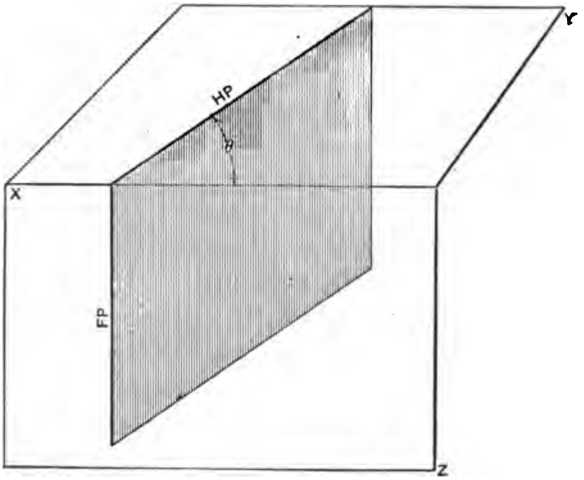


Fig. 26a.

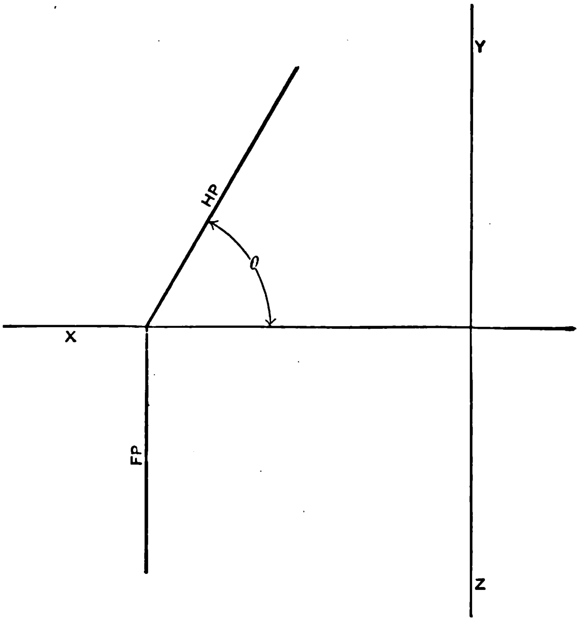


Fig. 26b.

the slope of the horizontal trace shows the angle between the plane P and the front plane. The same plane is shown by orthographic projection in Fig. 26*b*, and the angle marked θ measures accurately the same angle.

The problem is a little more complicated when the plane is oblique to both planes of projection as shown in Fig. 27*a*. In this case, of course, the angle between one of the traces and the X axis does not show the angle between the plane P and a plane of projection. If, however, an auxiliary plane, marked by its traces, $H\,A$ and $F\,A$, is placed so that it intersects the plane P, and is perpendicular to the horizontal plane as shown in Fig. 27*a*, the angle θ between the line of intersection of the planes P and A and the trace $H\,A$ is the angle between the plane P and the *horizontal* plane. The method of finding this angle from the *orthographic projections* is shown in Fig. 27*b*. The given plane is shown by its traces, $H\,P$ and $F\,P$; and the auxiliary plane by its traces, $H\,A$ (perpendicular to $H\,P$) and $F\,A$ (perpendicular to the X axis). The line of intersection of these two planes is shown by its projections $i^h\,j^h$ and $i^f\,j^f$. This line when revolved into the horizontal plane about its horizontal projection, $i^h\,j^h$, as an axis, takes the position $i^h\,j'$ and shows the angle θ in its true size. A dimension line marked s is shown to make the construction plainer. It should be evident that the auxiliary plane, A, can be drawn through any point in the trace of the given plane.

Similarly we can find the angle between the same plane, P, and the front plane by drawing the front trace, $F\,B$, of an auxiliary plane, B, perpendicular to the front trace of P, and making the horizontal trace, $H\,B$, perpendicular to the X axis. The required angle between the plane, P, and the front plane of projection is shown by the true size of the angle between the front plane and the line of intersection of the auxiliary plane, B, with the plane P.

46. The reverse of this last operation is made use of in drawing the traces of planes when only one of them is given on the drawing.

If the horizontal trace of a plane is given and the angle the plane makes with the horizontal plane is known, the front trace can

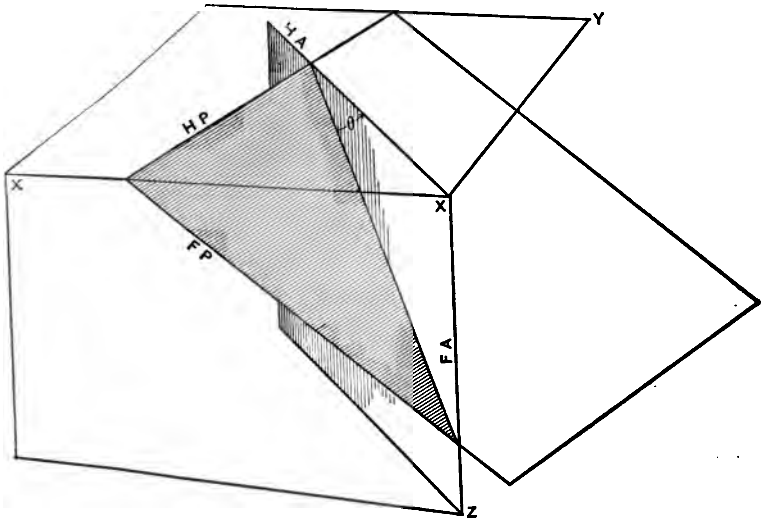


FIG. 27*a*.

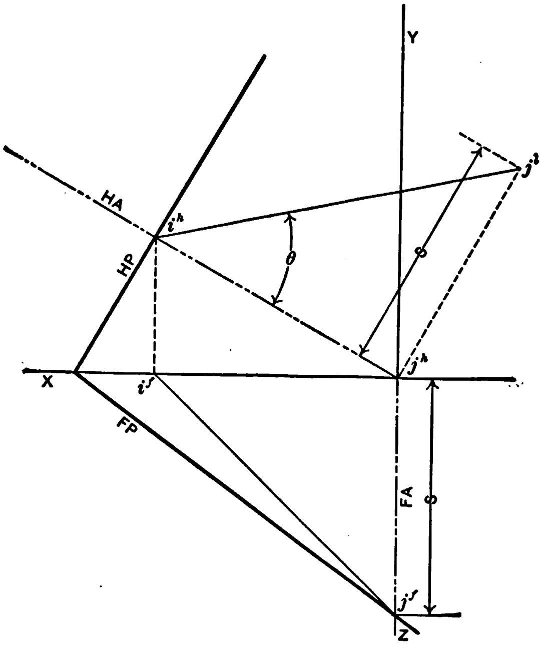


FIG. 27*b*.

be constructed. Assume now that in Fig. 27*b* the horizontal trace of the plane *P* and the angle, θ, the plane makes with the horizontal plane are given. Through any point in *H P* draw the horizontal trace, *H A*, of an auxiliary plane, and the line $i^h j'$ making with *H A* an angle equal to the angle the plane makes with the horizontal plane. This last line is the revolved position in the horizontal plane of a line $i\ j$ of which we know the horizontal projection, $i^h j^h$, and the distance (s) of the point j below the horizontal plane. The front projection of j is located by laying off on a line perpendicular to the X axis through j^h the distance marked s. The line $i\,j$ determined now by two projections is a line in the required plane, and j^f is a point in its front trace. The required front trace is drawn then through the projection j^f and the intersection of the given horizontal trace with the X axis.

The method is exactly similar when the front trace and the angle the plane makes with the front plane are given, and the horizontal trace is to be constructed.

47. PROBLEM 15. *To find the angle made by a given plane with either the horizontal or front planes of projection.*

Method. To measure the inclination to the horizontal plane: Through *any* point in the horizontal trace of the given plane pass an auxiliary plane perpendicular to this trace. (It will be perpendicular to the horizontal plane.) The true angle between the horizontal plane and the line of intersection of the auxiliary plane with the given plane is the required angle.

To measure the inclination to the front plane: Pass an auxiliary plane perpendicular to the front trace. The true angle between the front plane and the line of intersection of the auxiliary plane with the given plane is the required angle.

EXERCISES

87. Find the angles between the plane *P* (0, 120°, 210°) and the horizontal and front planes of projection.

88. Take the data given for the skew bridge in Ex. 27 and

find the angle between the plane of the portal and the horizontal and front planes of projection.

48. Problem 16. *Given one trace and the angle a given plane makes with the corresponding plane of projection, to find the other trace.*

Method. Reverse the method of the preceding problem.

EXERCISES

89. The front trace of a plane, *Q*, makes an angle of 315° with the *X* axis. The plane itself makes an angle of 45° with the front plane. Draw the horizontal trace.
90. The horizontal trace of a plane, *R*, makes an angle of 30° with the *X* axis. The plane itself makes an angle of 45° with the horizontal plane. Draw *F R*.
91. The plan of a dry dock is shown by the points a (−4, −2, 0), b (−2, −4, 0), c (−1, −3, 0), and d (−3, −1, 0). The planes of the sides through $a\ b$ and $c\ d$ slope toward the middle of the dock, and make angles of 45° with the horizontal. Draw the traces of the planes of the two sides mentioned.

49. When none of the traces of a plane are given and we know only the angles the plane makes with the planes of projection, we have a more complicated problem than the last. In Fig. 28*a* a plane, *P*, is shown in a pictorial drawing. A sphere is also represented with its center on the *X* axis at *e*, and is tangent to the plane *P*, at *i*. An auxiliary plane, *A*, is represented as drawn perpendicular to the front plane, and passing through the point *i* and the center of the sphere, *e*. This auxiliary plane cuts a great circle from the sphere and a line *a b* from the plane *P*. This last line is *tangent* to the circle at *i*. It is evident that the angle α between the line *a b* and the trace *F A* shows the angle between the plane *P* and the front plane. If, then, the line *a b* is revolved about the trace *F A* into the front plane, its revolved position will be shown by $a\,b'$. Likewise if the circular section cut from the sphere by the auxiliary plane is also revolved into the front plane it will be a circle,

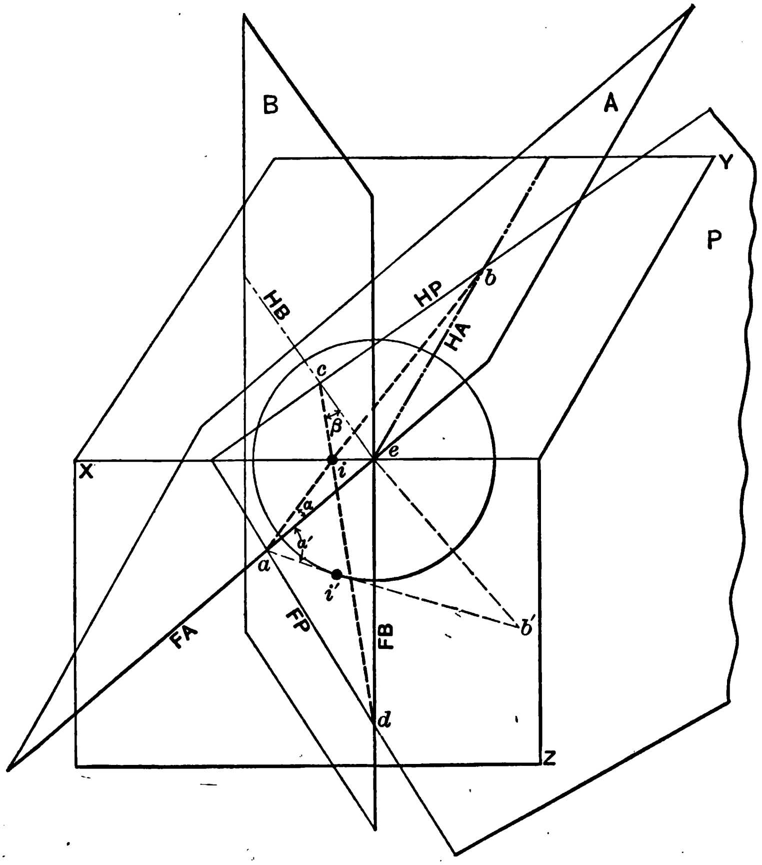


Fig. 28*a*.

coinciding, however, with the outline of the sphere. By this revolution into the front plane, the line $a\,b$ and the circular section of the sphere are shown in their true relative position, and the point of tangency, i, is shown at i'. The angle between $a\,b'$ and $F\,A$, marked α', is the true size of α.

Similarly an auxiliary plane, B, is represented in the figure, perpendicular to the horizontal plane and passing through the points e and i. This plane cuts from the given plane, P, the line $c\,d$ and a circular section from the sphere. The angle β between the line $c\,d$ and the trace $H\,B$ shows the angle between the plane P and the horizontal plane. By revolving $c\,d$ and the circular section into the horizontal plane the true size of β can be shown.

This method of analysis is most useful when the conditions are reversed—when only the angles α and β are given and the traces, $H\,P$ and $F\,P$, are to be determined. The actual process for the solution of this case is shown in Fig. 28*b*. Through a point e on the X axis as a center draw a circle of any convenient radius. This represents in orthographic projection the revolved position in either the horizontal or the front planes of any sections of the sphere cut by auxiliary planes passing through the center. In order to show the *revolved position of the line of intersection* with the plane, P, of an auxiliary plane perpendicular to the front plane, draw the line $a\,b'$ tangent to the circle at any convenient point, and draw a radius $e\,a$ so that the angle $b'\,a\,e$ is equal to the given angle, α, between the plane P and the front plane. The line $e\,b'$ is drawn perpendicular to $a\,e$. The triangle $a\,b'\,e$ that is thus formed corresponds to the revolved position of the triangle $a\,b\,e$ shown very much foreshortened in Fig. 28*a*. If now the length $e\,b'$ (Fig. 28*b*) is measured and laid off perpendicular to the X axis behind the point e, the point marked b^h is obtained. And this is a point in the horizontal trace $H\,P$. Continuing with the same general method, draw $c\,d'$ tangent to the circle at a convenient point, and by drawing the radius $c\,e$ lay off the angle β. The intersection of $c\,d'$ with $e\,d'$, perpendicular to $e\,c$, determines the point d', which is the revolved position of a point located on the trace $F\,P$. Laying off then $e\,d^f$ equal to $e\,d'$ the point d^f is obtained. If now $a\,e$ happened to

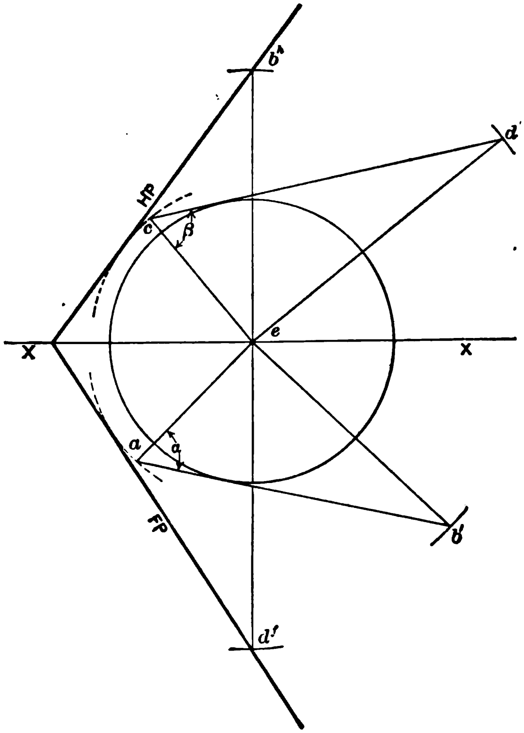


Fig. 28*b*.

be taken in such a position that it is perpendicular to *F P* in its true location, this trace could be drawn immediately through d^f and a; and, similarly, *H P* could be drawn through b^h and c. Since, however, *a e* was drawn in *any* convenient direction, it could not be taken necessarily perpendicular to *F P*; but, obviously, if an arc with a radius *a e* is drawn as shown in the figure, *F P* must be drawn tangent to this arc and through the point d'. For the same reason *H P* will be drawn tangent to the arc with a radius *c e* and through the point b^h.

50. PROBLEM 17. *To find the traces of a plane when the angles the plane makes with both the horizontal and front planes are given.*

Method. Imagine a sphere placed tangent to the plane of which the traces are to be found, and two auxiliary planes are passed through the center of the sphere and through the point where the sphere touches the plane—one perpendicular to the front plane, the other perpendicular to the horizontal plane. These planes will each cut a circle from the sphere and a tangent line from the given plane. The angles these tangents make with the front and horizontal planes, respectively, are the angles the given plane makes with the front and horizontal planes.

EXERCISES

92. Draw the traces of a plane, *P*, which makes angles of 45° and 60° respectively with the horizontal and front planes.
93. Draw the traces of a plane which is inclined 120° to the horizontal plane and 75° to the front plane.
94. The side of a bridge pier in a river makes an angle of 60° with the plane of the water, and 50° with a vertical plane at right angles to the course of the river. Represent the plane of the side of the pier by its traces on assumed planes of projection.

51. PROBLEM 18. *To find the true distance from a given point to a given line.*

Method. Determine the *plane* of the point and the line by

drawing through the point a line parallel to or intersecting the given line. Find the intersections of both lines with either the horizontal or the front plane. With a trace of this plane as an axis, revolve these lines into the corresponding plane of projection, and measure the distance required.

EXERCISES

95. Find the distance from the point *i* $(-2, -1, -1\frac{1}{2})$ to the line *j* $(-2, -\frac{1}{2}, -\frac{3}{4})$, *k* $(-\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{4})$.

96. The telephone wires running from a village to a house on the side of a mountain follow the shortest line between the points *v* $(-\frac{1}{2}, -1\frac{1}{2}, -2)$ and *h* $(-3, 0, -\frac{3}{4})$. A camp at *c* $(-2\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{4})$ is to be connected to the telephone system of the village by erecting a line joining the one between *v* and *h*. Draw the projections of the shortest line that can be put up; and find the length of the line thus located.

97 In the triangle given in Ex. 62 find the distance from the point *c* to the line *a b*.

98 For supporting a crane a cable is to be attached at one end to a point *c* $(-1, -1, -2)$ on its mast, and at the other end to a steel beam *a* $(-3\frac{1}{2}, -\frac{1}{2}, -2)$, *b* $(-1\frac{1}{2}, -2\frac{1}{2}, -1)$ on an adjoining building. Show the projections of the shortest cable that can be used.

52. PROBLEM 19. *To find the shortest distance between two lines not in the same plane.*

Method. Two lines *a b* and *c d* not in the same plane are shown in Fig. 29. Pass a plane, *M N*, through *a b* parallel to *c d* (Art. 19). Project *c d* upon this plane. At the point *e′* where the projection *c′ d′* intersects *a b*, draw a line perpendicular to the plane intersecting *c d* at *e*. The true length of the perpendicular *e′ e* is the required distance.

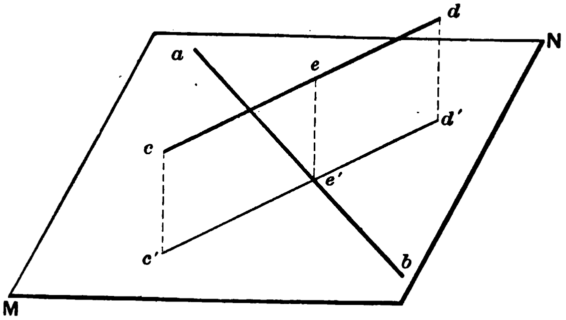


Fig. 29.

EXERCISES

99. Find the shortest distance between the lines $k\,l$ and $m\,n$ in Ex. 25.

100. Find the distance between a line from a to b and the mast of the derrick in Ex. 71.

101. Through each of two lines pass a plane parallel to the other line. Find the shortest distance between these two planes.

102. Solve Ex. 65 by the method of this article.

53. PROBLEM 20. *Given the projections of the center of a circle of known diameter, to draw its projections so that it shall lie in a given plane.*

Method. The center of a circle is shown by its projections c^h and c^f in Fig. 30 as lying in the given plane P. The horizontal and front projections of the circle are to be drawn so that the circle also shall lie in the plane. The center of the circle is revolved about $F\,P$ as an axis into the front plane, and the true size of the circle is drawn. When this circle is revolved back into the plane P, both projections will be ellipses, because the projections of a circle on planes of projection that are oblique to its plane are ellipses. The length of the major axis of the ellipse in the front projection is the same as the diameter of the circle and is, of course, parallel to $F\,P$. As all lengths which are perpendicular to the axis of revolution are foreshortened in the front projection, the minor axis is the forehortened diameter perpendicular to $F\,P$. The front projections of the minor axis is, then, on a line of indefinite length drawn through c perpendicular to $F\,P$. Mark the point where this projection crosses the front trace i^f, and revolve the line $i\,c$ into the front plane about $i^f\,c^f$ as an axis. The revolved position is shown by $i^f\,c'$, on which true lengths can be measured. From c' lay off the length $c'\,e'$ equal to the radius (r) of the circle. In counter-revolution the point c' revolves back to c^f and e' to e^f. Then $e^f\,c^f$ is the semiminor axis of the elliptical front projection of the given circle.

The horizontal projection of the circle is then easily found by

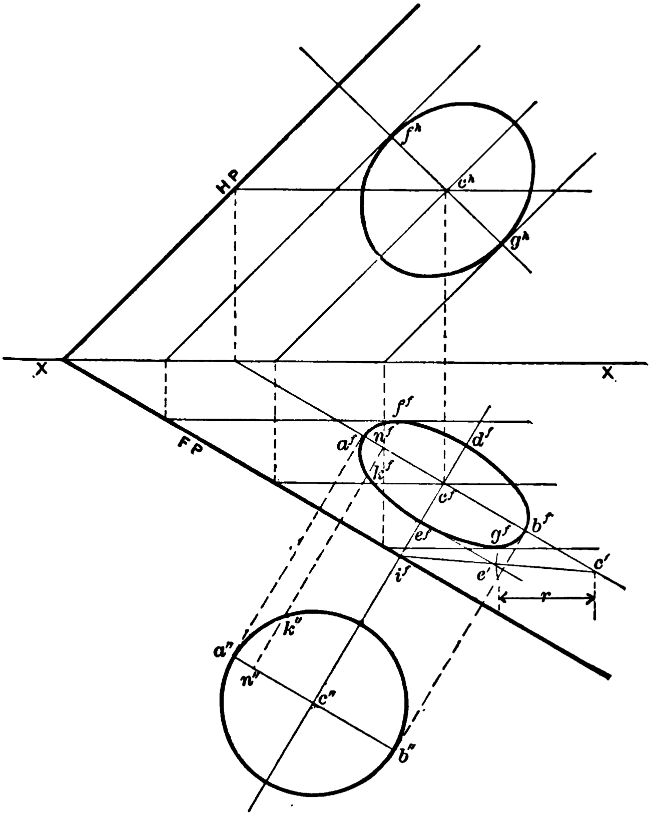


Fig. 30.

drawing lines that lie in the plane, P (such as diameters through the center, c, and tangents to the front projection) and *projecting* points on them from the front projection of the circle. The horizontal projection of the circle could be found also by repeating the method used for obtaining the front projection.

54. This problem has an important application in determining the plane of guide pulleys to direct belts running between pulleys which are on shafts at right angles to each other. In Fig. 31 two pulleys are shown with centers at a and b. The direction of motion is shown by arrows. The belt must be led off the pulley at b in *its* plane, and led on the pulley at a in *its* plane. To accomplish this, a guide pulley is needed to direct the belt. Usually the guide pulley may be placed at any convenient point between the two pulleys at a and at b. Select a point where the direction of the belt is to be changed, as the point d in the figure, and draw the lines $f\,d$ and $d\,e$ tangent to the pulleys. The plane of the guide pulley must be then in the plane of these two tangent lines. This plane is shown by the traces $H\,P$ and $F\,P$. The projections of the guide pulley are found by revolving the lines $f\,d$ and $d\,e$ into the horizontal plane of projection about $H\,P$ as an axis. The revolved position of d is shown at d'. The actual size of the guide pulley is shown tangent to the revolved positions of $f\,d$ and $d\,e$. After revolving back into the plane P, the *center* (c') of the guide pulley is shown by its projections c^h and c^f. The projections of the *guide pulley* are found by the method explained in the last article. The shaft of the guide pulley being perpendicular to its plane is shown by projections perpendicular to the corresponning traces.

In Fig. 31, on one side, the belt is led off the pulley a in *line* with the rim of the pulley b, so that on this side there is no change in the direction of the belt. In practice, however, very often the given pulleys are not placed so advantageously and two guide pulleys are needed.

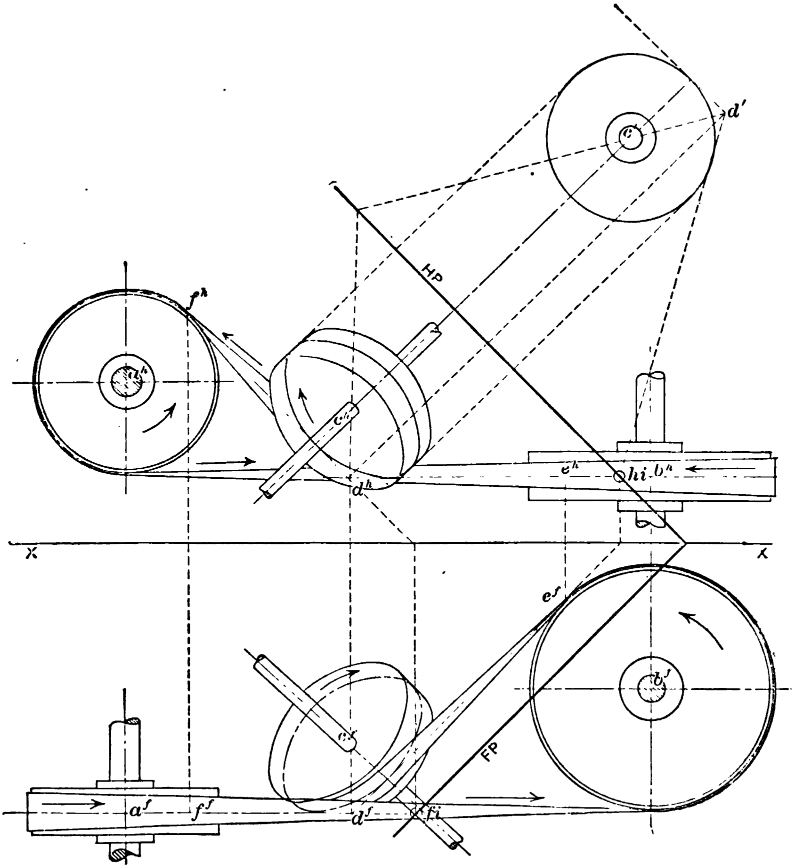


FIG. 31.

EXERCISES

103. Draw the projections of a circle, diameter 1½ inches, lying in the plane Q $(-3, 30°, 315°)$.

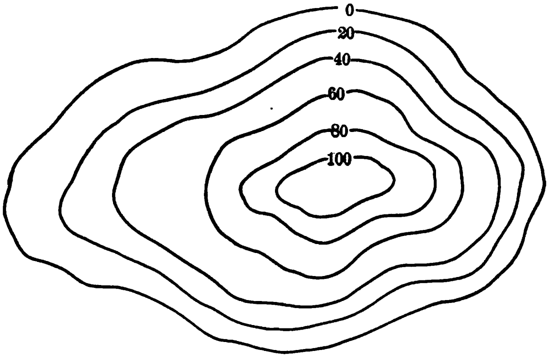
104. Two pulleys revolving on lines of shafting at right angles to each other are to be connected by a belt. Determine the planes of intermediate pulleys to properly direct the belt, drawing also the projections of all the pulleys that are needed.

55. PROBLEM 21. *Given a surface by its contoured * plan, and a plane by its slope, to determine the section of the surface cut by the plane.*

Method. Draw the horizontal projections of a number of horizontal lines lying in the given plane, and having the same indices as the contour lines. The points where these lines meet the contour lines with the same indices are points in the required section. The complete section is obtained by drawing a fair curve through the points thus obtained.

EXERCISES

105. The figure below represents a hill by its contour lines. Draw planes to show the embankments of a railroad "cut" passing through it. Show the intersections of these planes with the surface of the hill.



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* *Contour lines* are used to join points at the same elevation above a plane of which the elevation is assumed to be zero. Numbers on these lines, called *indices*, represent the elevations.

CHAPTER III

PROBLEMS RELATING TO PLANES TANGENT TO SOLIDS

56. Every surface may be generated by the motion of a line, and the different positions assumed by this line are called the **elements** of the surface.

A plane surface or a **plane** is generated by a straight line moving along another straight line and remaining always parallel to its first position.*

A **single curved surface** is generated by a straight line moving so that any two of its consecutive positions are in the same plane.

A **warped or twisted surface** is generated by a straight line moving so that **no two** of its consecutive positions are in the same plane.

Planes, single curved surfaces, and warped surfaces are generated by the motion of a straight line, and therefore all have elements that are straight lines. Every warped surface, however, is curved, and it is therefore possible to conceive it also as being generated by a curve which, as it moves, continually changes its form according to a definite law.

A **double curved surface** is generated by a curve moving along another curve. It has no elements that are straight lines.

57. A **cone** is a single curved surface generated by a straight line moving along a curve and also passing through a point not in the plane of the curve. This point is the **vertex** of the cone.

A **cylinder** is a single curved surface generated by a straight line moving along a curve with all its positions parallel. It may be regarded as a special case of a cone with the vertex at infinity.

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* A knowledge of the properties of planes was assumed in the beginning. The methods of representing planes, and points and lines in them, have been discussed in the preceding chapters.

Problems relating to cylinders are solved therefore by the same methods that are applied for similar problems relating to cones.

A plane cutting all the straight line elements of a cone, or of a cylinder, intersects it in a curve called the **base.**

If all the elements of a cone make the same angle with a straight line passing through the vertex, it is a **right cone;** otherwise, it is an **oblique cone.** If all the elements of a cylinder are perpendicular to the base, it is a **right cylinder;** otherwise, it is an **oblique cylinder.**

58. A **convolute** is a single curved surface generated by a straight line moving along a curve of double curvature, so that it is always tangent to the curve. The consecutive positions of the straight line generating the surface intersect two and two, no three intersecting in a common point.

There are as many kinds of convolutes as there are curves of double curvature. Some are important for their practical applications. The methods of constructing and representing them will be discussed in Art. 76.

59. A **surface of revolution** is one that is generated by the **revolution** of a straight line, or a plane curve about a straight line in the same plane as an axis.

There are only two single curved surfaces of revolution,—the right cone and the right cylinder, when they have circular bases. The principal double curved surfaces of revolution are the sphere, the ellipsoid, the torus, the paraboloid, and the hyperboloid—surfaces which are explained later.

60. A point in any plane or curved surface is determined by two projections. If, then, a *surface* is given by the projections of its outlines, and a *point* on the surface is given by only one projection, another projection can be located, after determining two projections of an *element* of the surface *passing through the point.*

In Fig. 32 a *cone* is shown by the horizontal and front projections of its outlines. A point *a* on the surface of the cone is given by its

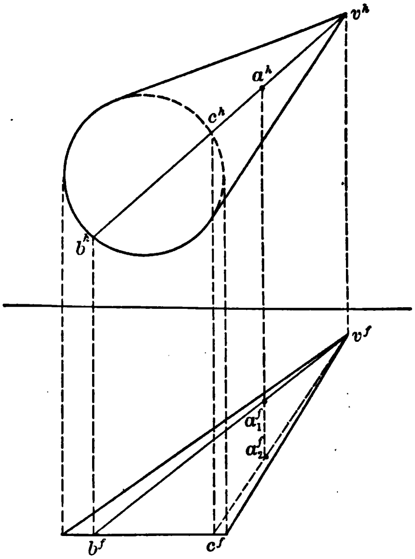

FIG. 32.

horizontal projection. The horizontal projection of an element of the cone is drawn through a^h and the vertex. As the point a may be on the top side of the cone nearest the horizontal plane, or on the lower side, there are two elements, $b\ v$ and $c\ v$, with horizontal projections passing through a^h. The front projections of these two elements, $b^f\ v^f$ and $c^f\ v^f$, are drawn through the vertex and through the front projections of the intersections of the two elements with the base. The required front projections of the point a are then on the elements $b^f\ v^f$ and $c^f\ v^f$, at $a_1{}^f$ and $a_2{}^f$.

For the case of the *cylinder* the same method of solution is applicable.

61. PROBLEM 22. *Given one projection of a point on the surface of a cone, to find the other projection.*

Method. Through the given projection of the point and the vertex of the cone draw elements of the surface. Find the required projections of the point on the other projections of these elements.

62. PROBLEM 23. *Given one projection of a point on the surface of a cylinder, to find the other projection.*

Method. Through the given projection of the point draw elements of the surface parallel to any element shown. As in the case of the cone preceding, the required projections of the point are found on the other projections of these elements.

63. When a point on a double curved surface, such as a sphere ellipsoid, torus, etc., is given by only one projection, a different method is used for finding the other projection of the point on the surface than that employed for the cone and the cylinder. In Fig. 33 the horizontal and front projections of an ellipsoid are shown. A point, a, on the surface of the ellipsoid is given by its front projection a^f. A plane, P, is drawn through a and the center of the ellipsoid perpendicular to the front plane. Its front trace is marked $F\ P$. If this plane is then revolved so that it is parallel to the horizontal plane, the front trace is shown by $F'\ P'$, and the front projection of a is at a^f. The plane $F'\ P'$ cuts from the ellipsoid a *section*, of which the horizontal projection of the surface is an

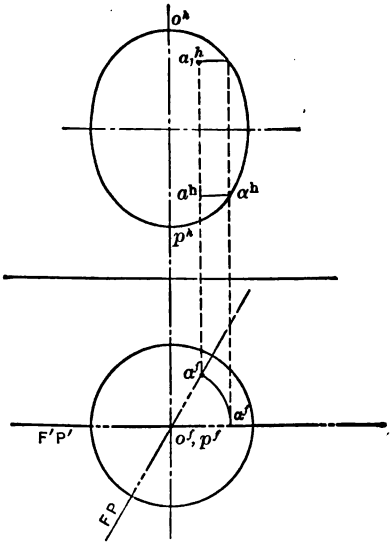

FIG. 33.

exact representation; and α (the revolved position of a) has its horizontal projection at α^h. In revolving the plane P, back to its original position, the point α moves to a in a circular arc lying in a vertical plane. The front projection of this arc is α^f a^f; and its horizontal projection is, of course, α^h a^h, parallel to the X axis. The required horizontal projection, a^h, is then determined by drawing projecting lines from a^f. Observe that the solution gives also another projection, a_1^h, shown at the top of the figure.

64. PROBLEM 24. *Given one projection of a point on a double curved surface, to find the other projection.*

Method. Through the given projection of the point and the center of the double curved surface draw the trace of a plane perpendicular to the plane of projection in which the projection of the point is given. Revolve this plane so that it becomes parallel to a second plane of projection. Determine the projections of the point in its revolved position. By revolving back to the original position, the required projection of the point is located.*

65. In general, a plane is tangent to a surface at a given point when it passes through tangents to two lines of the surface, meeting in the given point. If, then, through a given point, any two intersecting lines of the surface are drawn, and a tangent to each line is drawn at the point, the required plane is determined by the tangents.

For drawing a plane tangent to a single curved surface, the general method is somewhat simplified. In the case of the *cone*, we may observe that if a plane is tangent to the surface at a given point, it is tangent to the surface thoughout the length of an

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* Another method is to draw a right cylinder with its axis on the axis of the double curved surface, and with one of its bases passing through the projection of the point that is given. In Fig. 34 the point a is given by the projection a^f as in the preceding figure. Draw through a^f the front projection of the base of a right cylinder. The horizontal projections of the base are b_1^h c_1^h and b_2^h c_2^h, and of the point a are a_1^h and a .

If the projection of a point is given by its horizontal projection as d^h, its front projection is in an arc with the front projection of the axis of the surface as a center, and a radius equal to the horizontal distance from e^h to the outline of the ellipsoid. The front projections are then d_1^f and d_2^f.

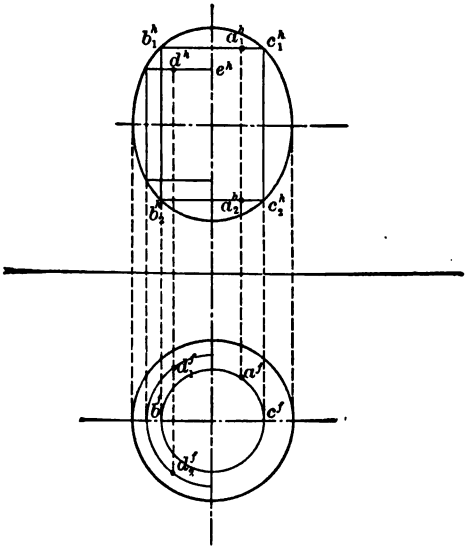


FIG. 34.

element passing through the given point, and is therefore tangent to the cone at the point where this element meets the base. The construction of tangents at the *given point* makes it necessary to represent sections of the cone, which is usually a laborious process. A tangent *to the base* at its intersection with the element is more easily constructed, and is used instead of the tangent to a section at the given point.

In Fig. 35 a cone is shown with a point, *a*, marked on its surface. The plane tangent to the cone at this point is determined by the element *b v* through *a* and by a tangent *b c* to the curve of the base at its intersection with *b v*. Two intersecting lines *b v* and *b c* are thus represented. They determine the plane, *P*, which is tangent to the cone at the point *a*.

As a cylinder may be regarded as a special case of a cone with the vertex at infinity, the problems relating to the cylinder will be solved by the same methods as for similar problems relating to the cone.

66. PROBLEM 25. *To pass a plane tangent to a cone or a cylinder through a given point on the surface.*

Method. Through the given point draw an element of the surface. At the intersection of this element with the base, draw a line tangent to the base. The required plane is determined by the element and this tangent line.

EXERCISES

106. Pass a plane tangent to an oblique cylinder through a point *b* on its surface.
107. The base of an inverted right circular cone is in the horizontal plane, and the angle between the elements of the surface and the axis is 30°. Pass a plane tangent to the cone through a point *c* on its surface.

67. A plane may be drawn tangent to a cone also through a point which is *outside* the surface. Both the vertex of the cone and the given point must be of course in the tangent plane. A line

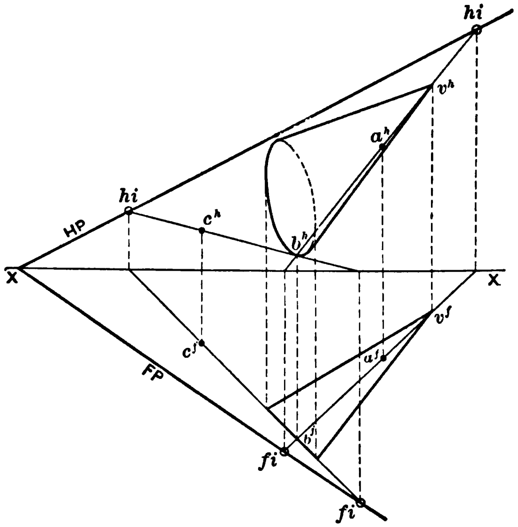


FIG. 35.

joining the vertex and the given point, and another line tangent to the base of the cone and intersecting the first line, determine the tangent plane.

In Fig. 36 a *cone* and a point *o* outside of it are shown. A line *o v* is drawn through the point *o* and the vertex *v*. Through the intersection of *o v* with the plane, *B*, of the base, the line *i t* is drawn tangent to the base. The tangent plane *P* is determined by these lines *o v* and *i t*.

When for the same conditions a *cylinder* is used instead of a cone, the solution is the same except that the line drawn through the vertex for the cone, is replaced by one through the given point parallel to an element.

68. PROBLEM 26. *To pass a plane tangent to a cone or a cylinder through a given point outside the surface.*

Method. Through the given point draw the projection of a line which, in the case of the cone, passes through the vertex, and, in the case of the cylinder, is parallel to an element. Produce this line to intersect the plane of the base and draw a tangent to the base through the point of intersection. The required plane is determined by this tangent line and the line already drawn through the given point.

EXERCISES

108. Pass a plane tangent to an oblique cone and through a point in the *X* axis.

109. Pass a plane tangent to a cylinder which has one base in the horizontal plane and another in the front plane, and *through* a point in the front plane outside the base.

69. PROBLEM 27. *To pass a plane tangent to a cone and parallel to a given line.*

Method. Through the vertex of the cone draw a line parallel to the given line. At the point where this line intersects the plane of the base, draw a line tangent to the base. These two lines determine the required plane.

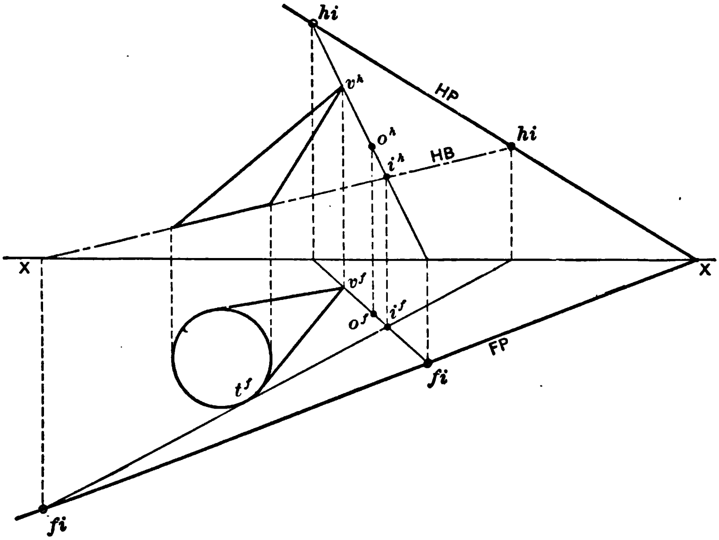


FIG. 36.

EXERCISES

110. Pass a plane tangent to an inverted oblique cone and parallel to an oblique line.
111. Pass a plane tangent to a right circular cone and parallel to a line in the side plane.
112. Pass a plane tangent to a right circular cone and parallel to an oblique line.

70. Problem 28. *To pass a plane tangent to a cylinder and parallel to a given line.*

Method. Through any point in the given line draw a line parallel to an element of the cylinder. The plane determined by the given line and the line just drawn will be *parallel* to the required plane. A plane tangent to the cylinder is then determined by a line tangent to the base and parallel to the plane already found, and the element of the cylinder which the tangent line intersects.

EXERCISES

113. Pass a plane tangent to an oblique cylinder and parallel to the X axis.
114. Pass a plane tangent to a right circular cylinder with its base in an oblique plane, and parallel to a line in the same oblique plane.

71. Problem 29. *To pass a plane tangent to a surface of revolution through a given point on its surface.*

Method. Draw through the given point a line tangent to the intersection of the surface with a plane passing through the point and the axis. When this line is revolved about the axis it generates a right cone tangent to the surface in a circumference which contains the given point. A plane tangent to the cone at the given point is the required plane tangent to the surface.

EXERCISES

115. Through any point, *a*, on the surface of an ellipsoid with a vertical axis, pass a tangent plane.
116. A circle 1 inch in diameter with its plane perpendicular to the horizontal plane, *revolves about a vertical axis* which is three inches from the center of the circle. A surface of revolution called a **torus** is thus generated. Through a point *b* on this surface draw a tangent plane.

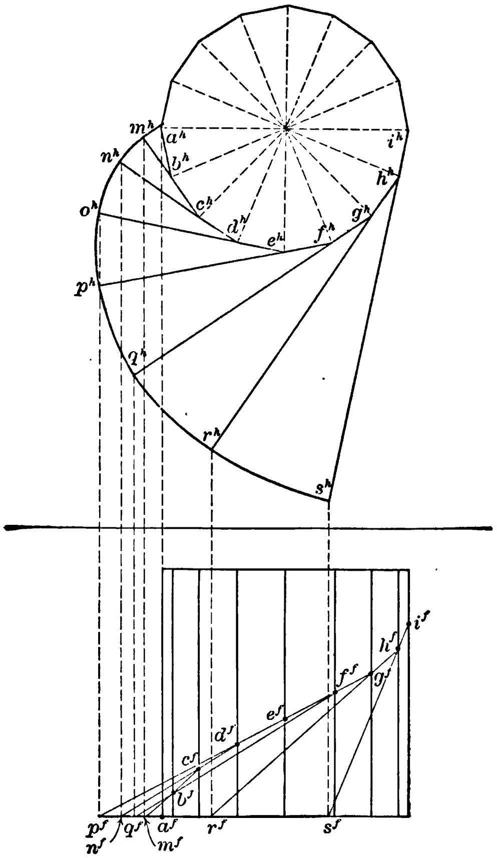
72. Problem 30. *To pass a plane tangent to a sphere and through a given line.*

Method. Pass an auxiliary plane perpendicular to the given line through the center of the sphere. (This plane cuts a great circle from the sphere and will cut a line tangent to this circle from the required tangent plane.) Revolve the auxiliary plane with its intersections with the given line and the sphere into a plane of projection. Through the point of intersection of the given line with the auxiliary plane draw a line tangent to the circular intersection of the auxiliary plane with the sphere. A plane passed through the tangent line (when revolved to its original or true position) and the given line is the one required.

EXERCISES

117. Draw a plane tangent to a sphere and passing through any oblique line.
118. Pass a plane through a line parallel to the X axis and tangent to a sphere with its center in the horizontal plane.

73. Convolutes. In Fig. 37 a regular prism with sixteen sides is shown. The student must now form a mental picture of a right triangle made of thin cardboard, resting on the projection $i^h\ s^h$ in the figure. This projection shall be also the horizontal leg of the triangle. The vertical leg is shown in the front projection of the prism by the distance from i^f to the base. We can imagine then such a paper triangle wrapped around the prism with the ver-



m^h
a^h
n^h
b^h
i^h
h^h
o^h
c^h
d^h
e^h
f^h
g^h
p^h
q^h
r^h
s^h
i^f
h^f
f^f
g^f
e^f
d^f
c^f
b^f
p^f
q^f
a^f
r^f
s^f
n^f
m^f

FIG. 37.

tical leg through *i* held stationary and the long leg of thc triangle always touches the base of the prism. The front projection of the hypothenuse of this paper triangle is then a broken line connecting the front projections of the points *i, h, g, . . . c, b, a.* Each portion of this broken line is **equally inclined** to the edges of the prism. The horizontal projection of the hypothenuse is the broken line joining the horizontal projections of the same points. The surface of the paper triangle is now unwrapped, taking it off one side of the prism at a time. And if it is turned on each edge of the prism in succession as on a hinge, till each unwrapped portion coincides with the plane of the next side, the portion of the hypothenuse that has been released becomes an extension and a tangent of the portion of the broken line on that side of the prism. Thus when the portion between *a* and *b* is unwrapped, it becomes tangent to the side in which *b c* lies; and when also the portion between *b* and *c* is unrolled, the portion *a b c* of the hypothenuse is tangent to the side containing *c d.*

Let us now consider the properties of this broken line from *a* to *i.* The portion *a b* intersects *b c, b c* intersects *c d,* etc.; that is, they intersect "two and two"; but *a b* does not intersect *c d* nor does *b c* intersect *d e,* etc.

74. Consider also another property of the broken line on the prism. If the number of sides of the prism is increased indefinitely, the broken line on its surface becomes a curve called a **helix.** It is a curve generated by a point moving on the surface of a cylinder of revolution so as to cut all the elements at a constant angle. The moving point has uniform motion **around**, and at the same time **parallel,** to the axis of the cylinder. The method for constructing a helix is shown in Fig. 38. The axis of the helix is vertical, so that the horizontal projection is a circle with its center at *o* in the axis. Now if the generating point moves the distance from m' to v' in making one complete revolution about the axis, passing through the points *m, p, s, t, u,* and *v,* the vertical distance between m' and v' is called the **pitch** of the helix.

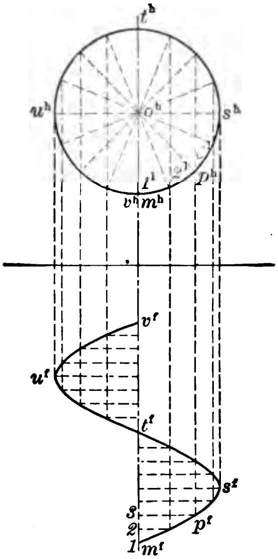


FIG. 38.

75. A **curved surface** is formed of the tangents to the broken line in Fig. 37 when the number of sides of the prism has been increased indefinitely, and it becomes a cylinder. The elements of this surface are the lines *m b*, *n c*, *o d*, *p e*, *q f*, etc. Such elements when tangent to a **cylinder** form a **convolute surface.** Portions of such surfaces are represented in Figs. 39 and 40. They are surfaces generated by a **straight line moving along a curve of double curvature, so that the line is always tangent to the curve.** In this surface, again, any two, but no three, consecutive straight line element lie in the same plane.

The convolute surface in Fig. 40 can be regarded as formed by the consecutive positions of the hypothenuse of a paper triangle as it is unwound from the surface of a cylinder of which the base is the circle a^h d^h n^h e^h. The point *a* at the end of the unwinding triangle will always lie in the curve a^h c^h i^h f^h in the horizontal plane. This curve is the **involute** * of the circular base of the cylinder; but it is also the horizontal trace of the convolute surface. The cylinder itself is no part of the surface. Observe the striking resemblance between the way this surface winds around a cylinder and the convolutions of a sea-shell.

When a helical convolute is to be represented on a drawing the curve of the helix to which the surface is tangent should be accurately constructed,—drawing first the top and front views of the cylinder on which it lies. If the axis is vertical, with the base in the horizontal plane, it is shown above that the horizontal trace of the convolute is the involute of the circular base.

76. If a point *m* on the convolute surface in Fig. 40 is given only by the horizontal projection m^h, the front projection can be found by construction. The horizontal projection of an element

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* If a tangent rolls upon a fixed curve, any point on it describes a second curve called an involute of the first. In Fig. 40 suppose a thread to be wound on the circular base and kept taut as it is unwound from the end of the thread at *a*, then the end at *a* will describe the curve $a^h c^h i^h f^h$, called the *involute* of the circle.

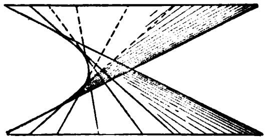


FIG. 39.

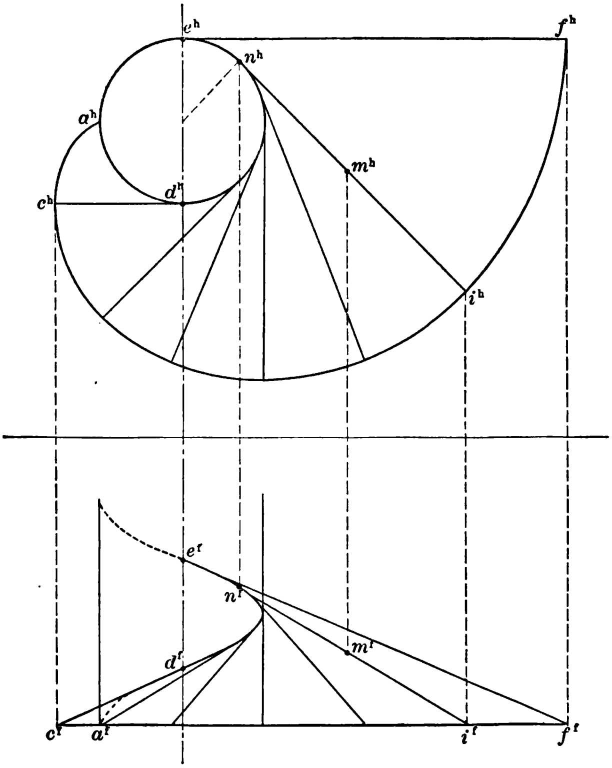


FIG. 40.

passing through the point *m* can be drawn tangent to the circular base at n^h, and it will intersect the involute at i^h. The front projections of *n* and *i* are respectively in the helix at n^f, and at i^f. By drawing the projection $n^f\ i^f$ and a projecting line from m^h, the front projection of *m* is determined.

The solution can be reversed: If the front projection of a point on the convolute surface is given, the horizontal projection can be found.

77. **Problem 31.** *Given one projection of a point on a convolute surface, to find the other projection.*

Method. Draw a projection of an element through the given projection of the point. Determine the other projection of this element and locate on it the required projection of the point.

EXERCISES

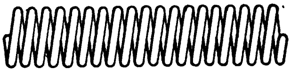
119. Draw the projections of ten equidistant elements of a helical convolute surface with a vertical axis. The diameter and pitch of the helix are respectively 2 inches and 3 inches.
120. Find the intersections of the convolute surface given in Ex. 119 with two planes perpendicular to the axis of the helix. Observe the shape of the curves cut by these planes.
121. Draw the projections of six elements of a convolute surface of which the axis is parallel to the *X* axis. The diameter and pitch of the helix are respectively 2½ inches and 4 inches. Assume one projection of a point on the surface and locate the other projection. (Use the side plane.)
122. Draw two helices, one right-handed and the other left-handed, on a cylinder 1½ inches in diameter and 2 inches high, the former to be of 1 inch pitch and the latter of ½ inch pitch.
123. Represent a square-threaded screw of the following

dimensions: Outside diameter of the thread is 2½ inches. Diameter at the bottom of the thread is 1½ inches. Pitch is 1 inch. Thickness of the thread is ½ inch. Show two complete turns of the thread.

124. A spiral spring is of the form of a square screw thread. The cross-section is ½ inch square, and the outside diameter and pitch are respectively 3 inches and 2 inches. Draw two complete turns of the spring.

125. A spiral spring is made of round wire as shown in the figure below. The diameter of the wire is ½ inch. The outside diameter of the spring is 2½ inches and the pitch is 2 inches. Draw the plan and elevation of two turns of the spring.

Suggestion. If a spiral spring is made of round wire, we conceive its surface to be generated by a sphere moving along a helix which is the center line of the wire. The projections of the helix are first drawn and then the projections of the sphere in a number of different positions.



78. Problem 32. *To pass a plane tangent to a convolute surface through a given point on the surface.*

Method. (The same as for the cone or the cylinder.) Through the given point draw an element of the surface. At the intersection of this element with the base draw a tangent. The required plane is determined by the element and the tangent line.

79. Problem 33. *To pass a plane tangent to a convolute surface and through a given point outside the surface.*

Method. Through the given point pass a plane which is perpendicular to the axis of the surface. The intersection of this plane with the surface is an involute, to which a tangent is then drawn from the given point. Draw an element of the surface at the point

of tangency. This element and the tangent line determine the required plane.

EXERCISES

126. Draw a plane tangent to the helical convolute constructed in Ex. 119, at any point on the surface.
127. Draw a plane tangent to the convolute surface constructed in Ex. 121 and through any point outside the surface.

CHAPTER IV

INTERSECTIONS AND DEVELOPMENTS OF SOLIDS

80. The **intersection of any surface with a given plane** is found by passing a series of **auxiliary planes,** in such a way that they cut from the surface, **straight lines, circles or other curves that can be quickly drawn**; and from the given plane, **straight lines.** The intersections of these lines cut from the surface, with the lines cut from the plane, determine points on the required intersection. When straight line elements can be cut from the surface the auxiliary planes are usually passed perpendicular to one of the planes of projection. When circles can be cut from the surface, the auxiliary planes should be drawn parallel to the plane of projection on which the projections of the circles will appear in their true form.

The **tangent to a curve of intersection** at a given point on a surface lies in the plane which cuts from the surface this line of intersection. It is also a line in the tangent plane to the surface at the given point.

81. The curve of intersection of an **oblique cone** with a plane P is shown in Fig. 41. The required curve of intersection is shown in the figure by its elliptical projections on which the points s and t are marked. The curve is obtained by drawing a series of **auxiliary planes through the vertex of the cone and perpendicular to the front plane** (the horizontal traces will be perpendicular to the X axis). Each auxiliary plane cuts from the cone **elements,** and from the given plane a **straight line.** In the figure, one of these auxiliary planes is marked by its traces $H\,A$ and $F\,A$. This auxiliary plane cuts from the cone the elements $a\,v$ and $b\,v$ and from the plane a straight line $m\,n$. The intersections of the projections of this last line with the projections of $a\,v$ and $b\,v$ give respectively the points s and t on the required line of intersection.

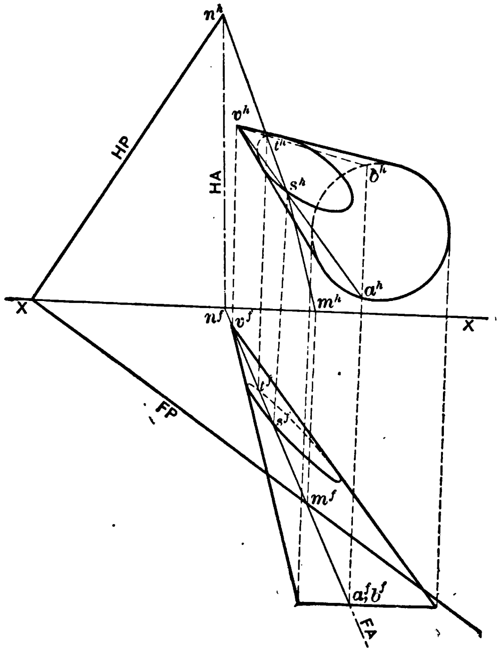


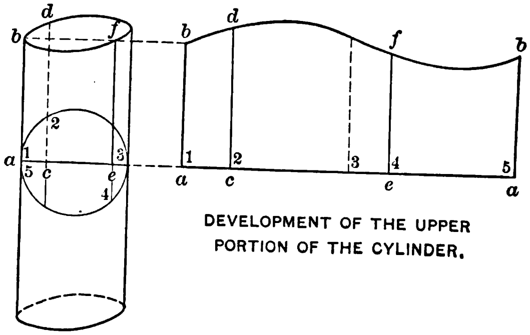
Fig. 41.

The curve of intersection of an oblique cylinder with a given plane is found by the same general method, using auxiliary planes that will cut elements from the surface of the cylinder and straight lines from the given plane.

82. A plane which is tangent to a given cone or a cylinder, contains an element of its surface. If, then, the surface of a cone, or a cylinder, is rolled on a tangent plane until each of its elements has come into this plane, the part of the plane passed over and included between the extreme elements is a **plane surface equal to the given surface.** The surface thus passed over is called a **development.** In order to determine the positions of the different elements of the surface as they come into the plane of development, it is necessary to **locate some curve on the surface which will develop into a straight line or a simple curve,** upon which the actual distances between the elements can be laid off.

In Fig. 42 an oblique cylinder is shown. The section cut from this cylinder by a horizontal plane happens to be a circle. Through the point *a* a plane is passed, cutting the surface in the straight line through the points *a*, *c*, and *e*. The curve of intersection when revolved through 90° into the plane of the drawing is shown as the circle 1, 2, 3, 4, 5. **The circumference of this circle is the actual length of the development of the cylinder.**

If we wish to develop the part of the cylinder above the intersection, we draw the base line *a a*, making its length equal to the circumference of the circular intersection. A point, *b*, on the curve of the development is immediately located over the point *a*, at a distance laid off on an element, equal to that from *a* to *b* in the top base. Other points are taken in the same way. To locate another point on the curve of the development, the distance 1, 2 is laid off on the base line, equal to the arc 1, 2 and the length *c d* is the width at this point of the surface we are developing. By continuing the process a series of points is obtained, and the surface included between the curve drawn through these points and the base line is the development of the surface of the cylinder above the intersection with the plane passed through *a*.



DEVELOPMENT OF THE UPPER PORTION OF THE CYLINDER.

Fig. 42.

The method for the development of a cone being similar, it needs no explantion.

The problems relating to developments of surfaces are constantly **applied by workers in sheet metal.** Of course, theoretically, it makes no difference which element of the surface is cut for the development; but, practically, it is economical "to cut so as to make the shortest seam," unless, however, there are good reasons for doing otherwise.

83. PROBLEM 34. *To find the intersection of any cone with any plane.*

Method. Pass through the cone a series of auxiliary planes taken either perpendicular to its axis, or through the vertex and perpendicular to one of the planes of projection. For the curve of intersection join the points where the lines cut from the cone intersect the lines cut from the plane.

EXERCISES

128. Draw the curve of intersection of a right cone, axis vertical, with an oblique plane. Draw the *development* of the portion of the cone between the line of intersection and the vertex.

129. Taking the data of Ex. 42, determine the curve of intersection of the cone of rays of maximum intensity with the plane of the desk.

130. Make the drawings for the patterns of the bath-tub shown in the figure on page 134.

    Suggestion.—This exercise requires merely the development of portions of two cones with vertices at *n* and *o*, and the patterns of plane surfaces. Assume suitable dimensions for those not given in this exercise, and in those that follow.

131. Draw a pattern for the sheet metal for a regularly flaring (*conical*) roof connection like the one shown in the

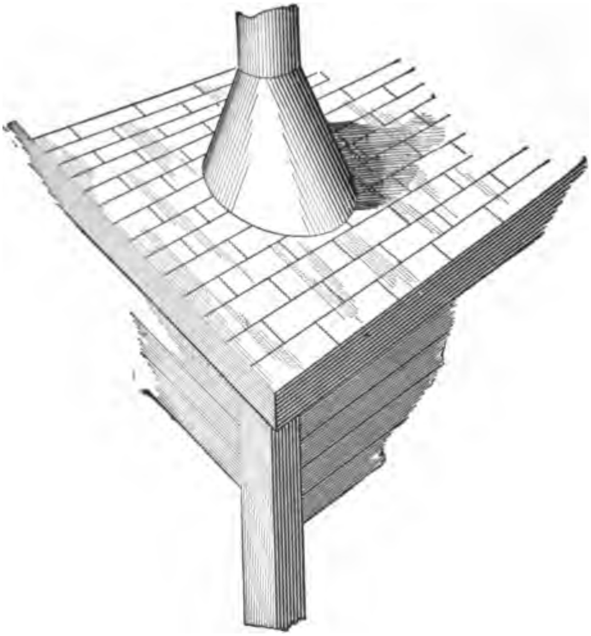
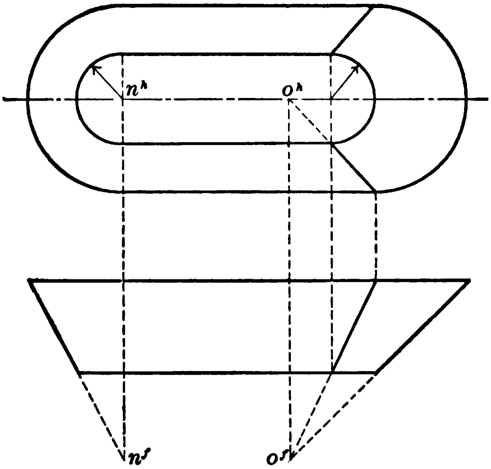


figure. The axis of the *cylindrical* pipe is vertical and the plane of the roof is represented by $R(0, 30°, 315°)$.

132. A conical tower is to be placed on the right-hand side of the irregular hip roof shown in Ex. 82. Show the true-size of the hole to be cut in the roof.

Suggestion.—Find the true size of the curve of intersection by *revolving the curve* about one of the traces of its plane into a plane of projection. Cf. Art. 53.

133. Draw the pattern for a "boot" to join two pieces of stovepipe, one of which is circular and the other oval. The circular pipe is 6 inches in diameter, and the oval pipe is represented by parallel sides, 2½ inches apart and 5½ inches long, with semicircles at the ends of the parallel sides. In other words, the over-all dimensions of the oval are 8 inches by 2½ inches. Make the "boot" 10 inches long.



84. Problem 35. *To find the intersection of any cylinder with any plane.*

Method. Pass through the cylinder a series of auxiliary planes, taken either perpendicular to its axis, or parallel to its axis and perpendicular to one of the planes of projection. For the curve of intersection join the points where the lines cut from the cylinder intersect the lines cut from the given plane.

EXERCISES

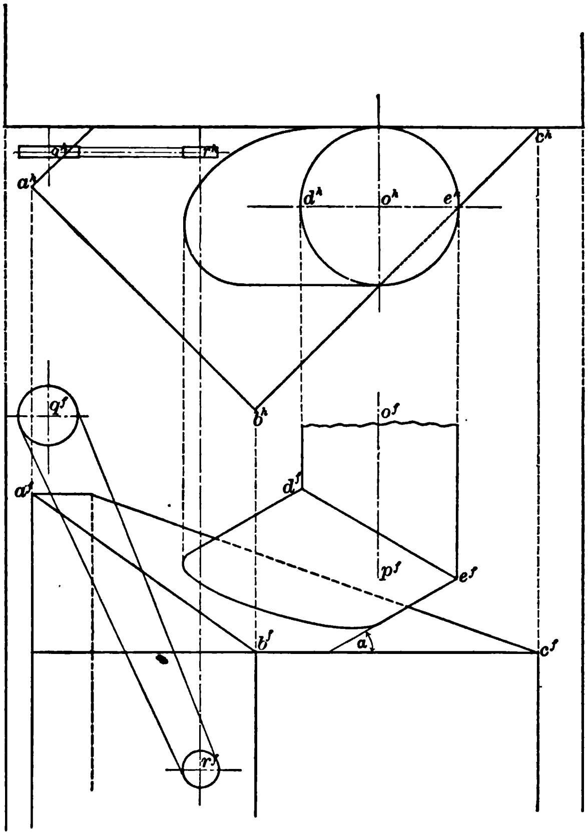
134. Find the true size and shape of the hole cut for a circular chimney in a tin covering for the right-hand side of the hip roof given in Ex. 79.

135. Draw the pattern for the inclined end of a bath-tub with a semicircular cross-section. (The inclined end is assumed to be a plane surface.

136. Find the true size of the opening to be cut in the *wall* shown in Fig. 51 for a pipe 3 feet in diameter, making angles of 30° and 20° respectively with the horizontal, and with the vertical plane of the back of the wall.

137. In the figure a flue from a boiler-house is shown passing through the roof of a small shed. Find the true

size of the hole that was cut in the roof for the flue. Make a pattern to show a sheet of tin to cover this roof with the hole cut out for the flue. Develop the inclined portion of the flue above the roof. (Select suitable data for the coördinates of the points that are marked in the figure.)



85. Problem 36. *To find the line of intersection of any surface of revolution with any plane.*

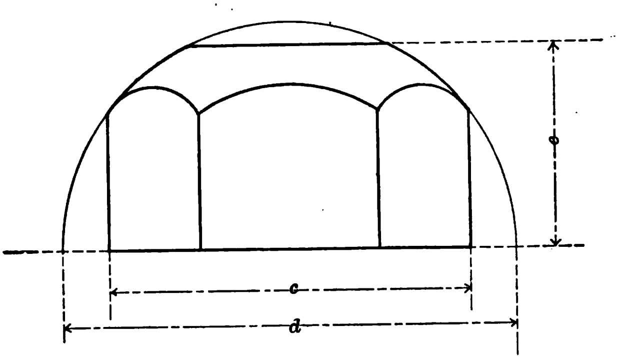
Method. Pass through the surface of revolution a series of auxiliary planes perpendicular to its axis. These planes cut circles from

the surface of revolution. The intersections of the circles cut from the surface with the lines cut from the plane determine the required curve of intersection.

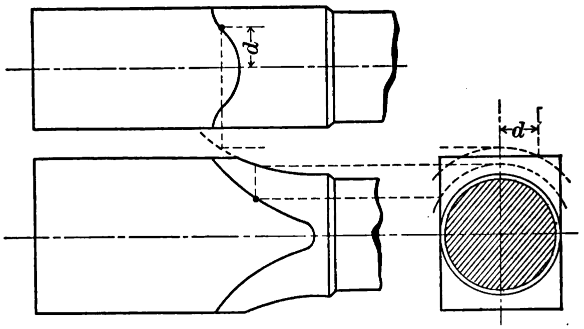
EXERCISES

138. Find the intersection of the torus given in Ex. 116 with an oblique plane.

139. Draw the projections of the hexagonal nut shown in the figure, representing accurately the lines of intersection.



140. Draw accurately the curves of the "stub end" of a connecting-rod.



141. The blades of a ventilating fan are plane surfaces attached to a spherical hub. Show the intersections.

86. The **intersection of any two curved surfaces** is found by passing auxiliary planes to cut from each surface lines that can be easily constructed. The intersections of these lines give points on the required curve of intersection. The auxiliary planes should be selected so that the lines cut from the surfaces are straight lines, circles parallel to a plane of projection, or some other curve of which the projections can be easily drawn.

87. A shaded drawing of two intersecting cones is shown in Fig. 43. The same cones are shown in orthographic projection in Fig. 44. The curves of intersection that are shown were obtained by the use of auxiliary planes passed so as to cut elements from each cone; in other words, the auxiliary planes were passed through the vertices of both cones. The elements cut from each cone by an auxiliary plane are drawn through its intersections with the base and through the vertex.

In Fig. 44 an auxiliary plane, *A*, is marked. Since it is a plane passing through the vertices of both cones, its horizontal trace, *H A*, is drawn through the intersection of the line *v w* with the plane of the bases, marked x^h. Observe that the horizontal projections of the cones are represented on the plane of the bases and that now the line through c^f, d^f, x^f is used as the axis showing the intersection of the horizontal and front planes. The auxiliary plane cuts from the cone with vertex *v*, the elements *c v* and *d v*; and from the cone with vertex *w*, the elements *a w* and *b w*. The intersections of the horizontal projections of these elements "two and two," determine the horizontal projections of four points on the curves of intersection. The curves in the front projection are obtained by drawing the projections of the elements of one cone, and projecting on them the points in the horizontal projections of the curves of intersection. The projections of two points in the curves are marked *m* and *n*.

88. PROBLEM 37. *To find the curve of intersection of two cones.*
Method. Pass through the cones auxiliary planes drawn through

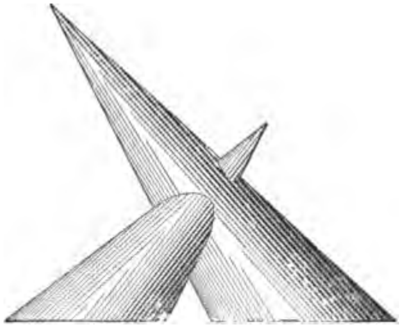


FIG. 43.

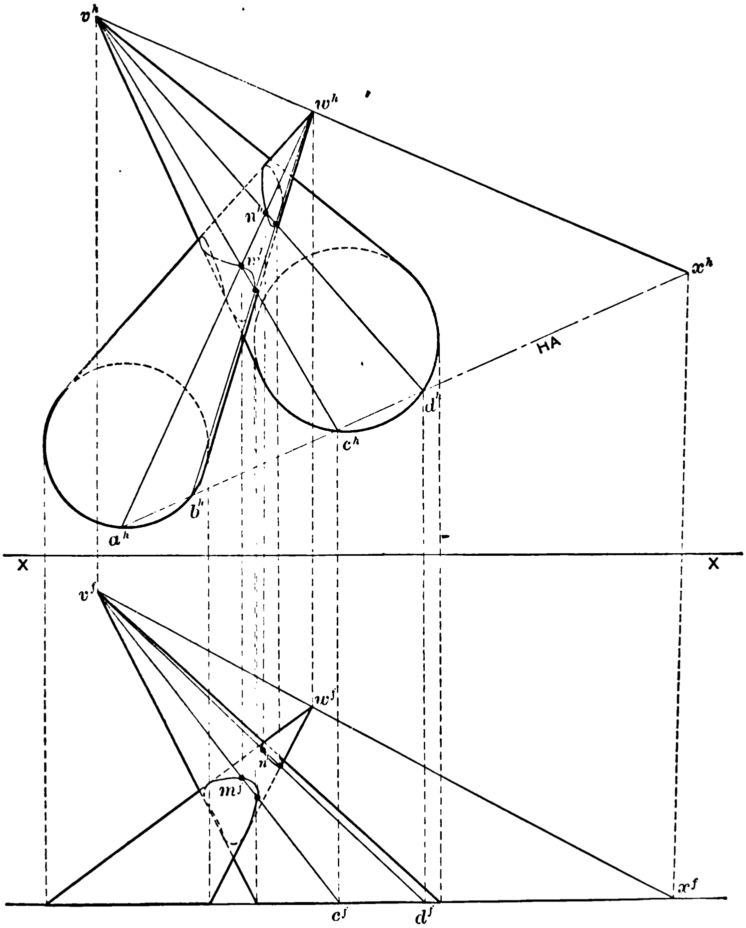


FIG. 44.

the line joining their vertices. Elements cut from each cone by these planes determine by their intersections the required curve.

EXERCISES

142. Find the curve of intersection of two right cones with axes that do not intersect. The axis of one cone is horizontal, the other vertical. Draw the development of the cone with the horizontal axis, showing the curve of intersection.

143. Draw the curve of intersection of two oblique cones with intersecting axes. Show the development of one of the cones.

89. In Fig. 45 the method most commonly used for finding the intersection of two cylinders is shown. The solution is a simplified method of passing planes perpendicular to the front plane—one set parallel to the axis of the smaller cylinder and another set parallel to the axis of the larger cylinder. The figure is lettered so that no other explanation is needed.

90. PROBLEM 38. *To find the curve of intersection of two cylinders.*

Method. Pass through the cylinders two sets of auxiliary planes, one set parallel to each axis. The elements cut from each cylinder by these planes determine by their intersections the required curve.

EXERCISES

144. Draw accurately the lines of intersection appearing on the surface of the flanged pipe fitting, shown in the figure on page 144. The fitting is made up of two cylinders with their axes intersecting each other symmetrically.

145. Find the intersection of two oblique cylinders with their bases in the horizontal plane.

146. A vertical steam-drum is to be put on a horizontal cylin-

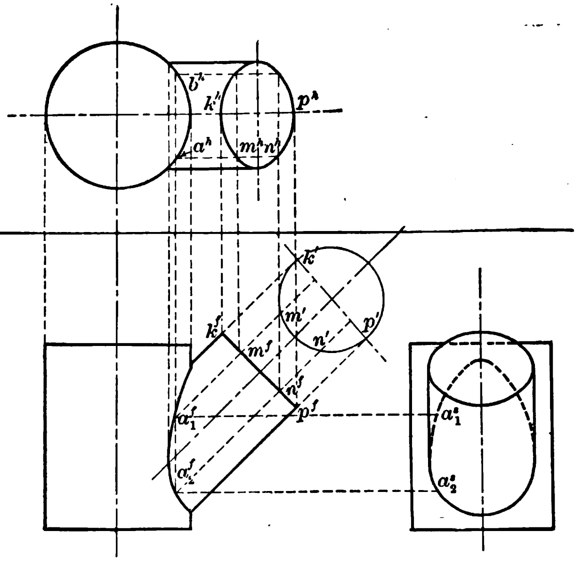
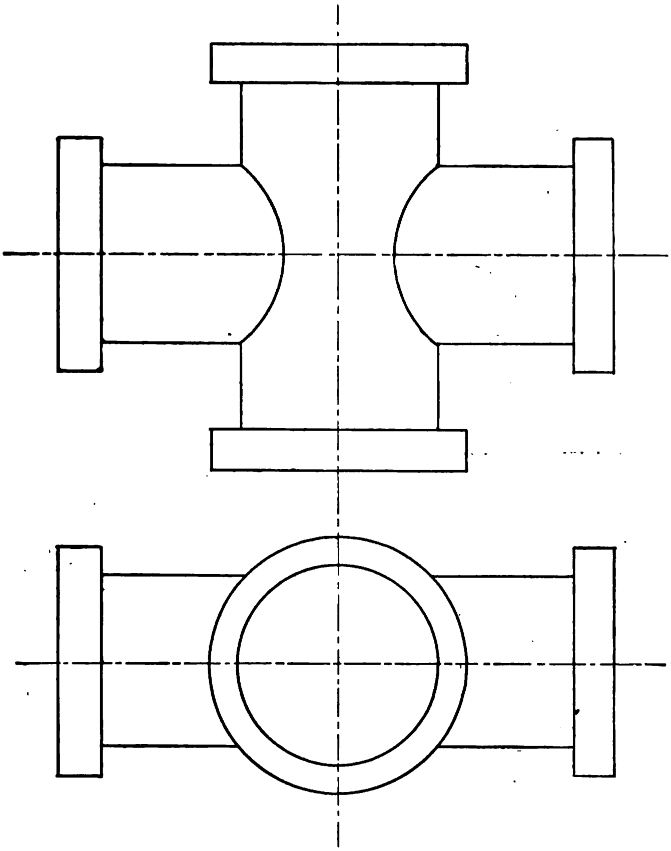


Fig. 45.

drical boiler. Make a pattern to show the size of the hole to be cut in the boiler plate. (Cf. Fig. 49.)



147. Find the intersection of two cylinders forming a branch "Y" for a blowpipe as shown in the figure on page 146. Develop both cylinders to show the curve of intersection.

91. In Fig. 46 a very simple case of the intersection of a cone and a cylinder is shown. The horizontal projection of the cylinder shows immediately the horizontal projection of the curve of intersection, and the front projection of the curve is found by projection.

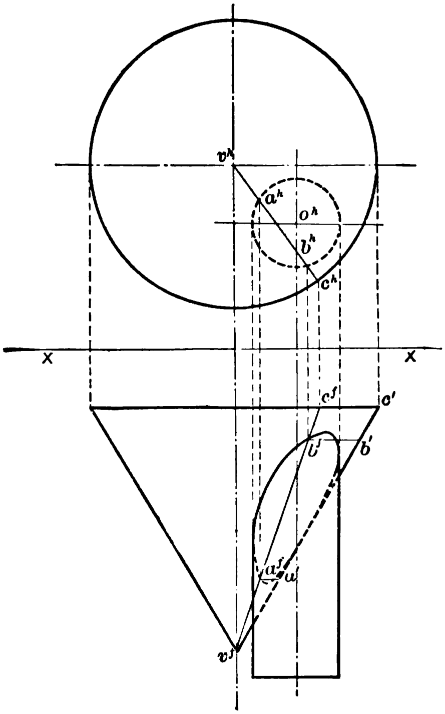
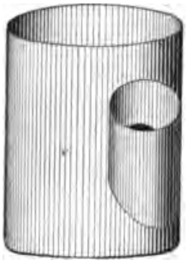


FIG. 46.

Horizontal lines are drawn through the front projections of the points *a*, *b*, and *c* to show a simple method for finding the true lengths of elements of the cone, and the true distances from the vertex of points on an element. These true lengths are needed for a development of the surface of the cone.



This is a special case. The general method is stated in the next *problem.*

92. PROBLEM 39. *To find the curve of intersection of a cylinder and a cone.*

Method. Pass through the vertex of the cone auxiliary planes parallel to the axis of the cylinder. The elements cut from each surface by these planes determine by their intersections the required curve.

When the two surfaces have circular bases, it is most advantageous to use auxiliary planes which cut circles from each surface.

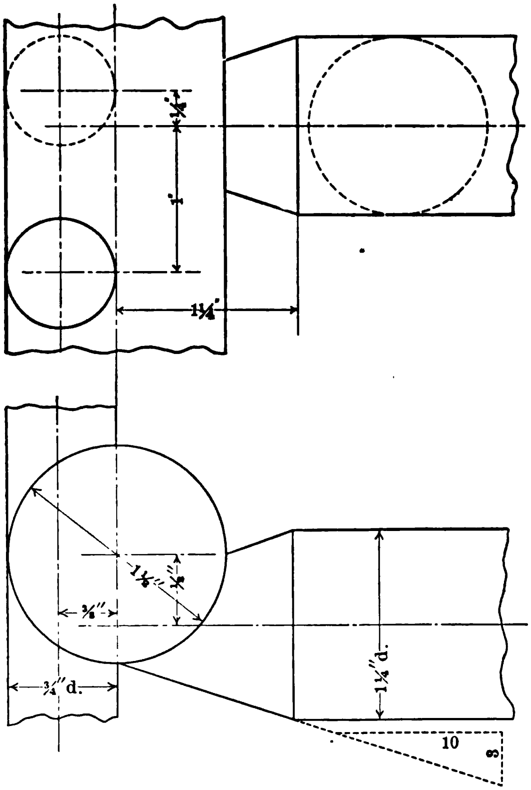
EXERCISES

148. A circular tower has a conical roof through which a horizontal pipe passes. Find the size of a covering for the roof and show in the development the hole cut for the pipe.
149. Find the intersection of an oblique cone with a right cylinder. The axes do not intersect, and one axis is parallel and the other perpendicular to the front plane.
150. Make the necessary patterns for an arch stone of a conical arch in a circular wall.

151. A horizontal steam pipe 1½ inches in diameter is intersected by a conical nozzle and two smaller vertical pipes as shown in the figure. The slope of the curved surface of the nozzle is $\frac{3}{10}$.

(*a*) Determine the developed true size of the hole cut in the horizontal pipe by the nozzle.

(*b*) Develop the conical surface of the nozzle.



152. Find the intersection of the cone and the cylinder forming the steam exhaust-head shown in the figure on page 150.

93. PROBLEM 40. *To find the line of intersection of any two surfaces of revolution (sphere, ellipsoid, torus, etc.).*

Method. If the axes of the two surfaces of revolution intersect.

the point of intersection of the axes is taken as the center for a series of auxiliary spheres. These planes cut circles from each surface. The intersections ot these circles with each other are points on the required curve.

If the axes do not intersect, pass through the surfaces auxiliary planes perpendicular to the axis of one surface and cutting circles from that surface, and cutting some other curve from the other surface. The intersections of the corresponding projections of the curves give the points to be found.

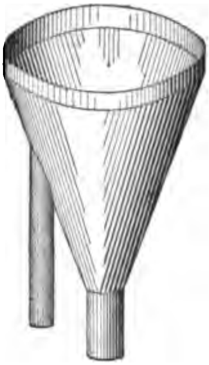


Fig. 47 shows the intersection of a sphere with an ellipsoid. The point *m* is taken as the center for the auxiliary spheres (at the intersection of the axes). The arcs of the auxiliary circles shown determine by their intersection the point x'. By continuing this construction a series of points is obtained which when connected by a smooth curve gives the line of intersection $a'b'$. The horizontal projection is obtained easily by projection.

EXERCISES

153. Draw the curve of intersection of a sphere and an oblique cone.
154. Draw the line of intersection of the ellipsoid and the torus shown in the first figure on page 152.

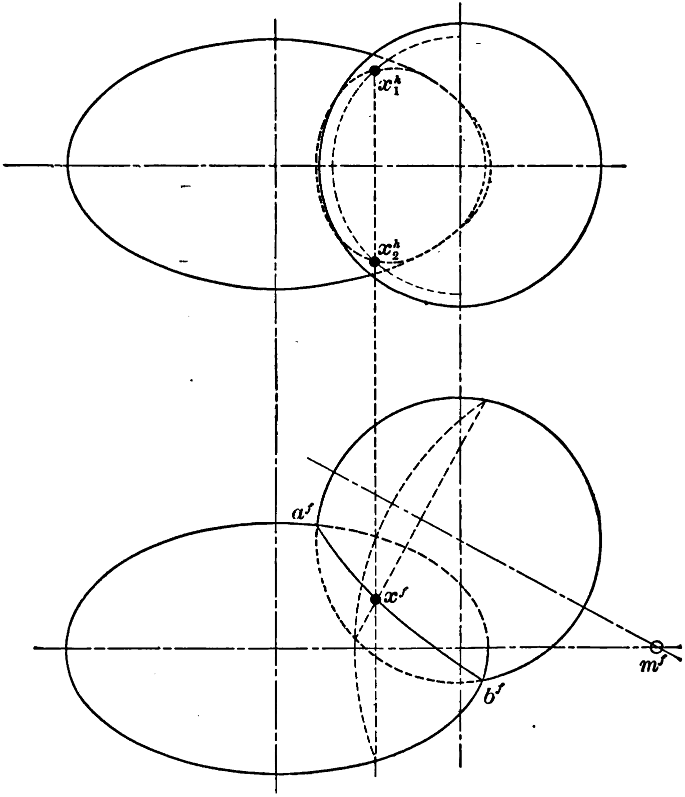
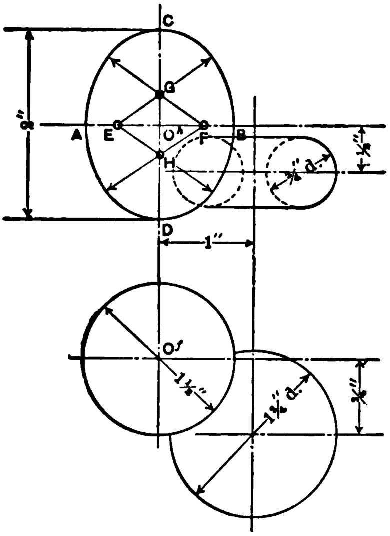
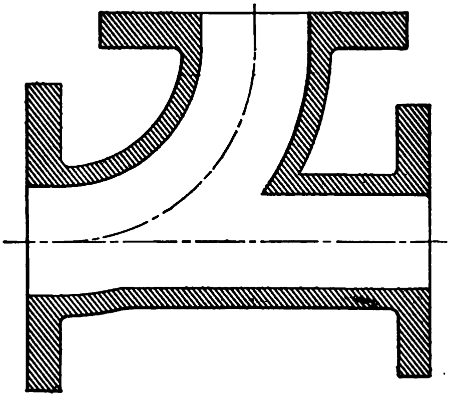


Fig. 47.

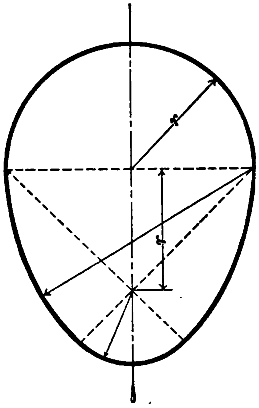


Suggestion. Use the approximate method of circular arcs for constructing the ellipse in the figure, making $CD - AB = EO^h = O^hF$ and $GO^h = O^hH = \frac{3}{4}EO^h$.

155. A pipe fitting is shown in the figure below. The center line of one portion is the arc of a circle intersecting the axis of the smaller portion which is cylindrical. Make the diameters respectively $1\frac{1}{2}$ inches and 1 inch. Draw the curves of intersection on the outside and inside of the fitting.



156. The sewer shown in the figure below by its section is intersected by two elliptical sewers with major and minor axes of 4 feet and 2½ feet. The dimension marked *r* for the large sewer is 5 feet. Show in a *plan* drawing the lines of intersection to be made in the masonry where the sewers meet. The bottoms of the three sewers are in the same plane. The small sewers are on opposite sides of the large sewer.



CHAPTER V

MISCELLANEOUS PRACTICAL EXERCISES

157. A shaft of a mine follows approximately the line between the points *a* $(-3, -\frac{1}{2}, -1\frac{1}{2})$ and *b* $(-1, -1\frac{1}{4}, -\frac{1}{8})$. A tunnel is to be made from a point *c* $(-2\frac{1}{4}, -1\frac{1}{2}, -\frac{1}{2})$ on the side of a mountain to intersect the shaft. Find the shortest length of the tunnel, and the angle between the center line of the shaft and the center line of the tunnel.

158. In Fig. 48 a channel and an I-beam are shown as they intersect in a roof. Take the following coördinates for the points marked in the figure: *a* $(+1, -1\frac{1}{2}, -3\frac{1}{2})$, *b* $(+3\frac{1}{2}, -4, -1\frac{3}{16})$, *c* $(+5\frac{1}{2}, -4, -\frac{1}{4})$, *d* $(+2, -\frac{1}{2}, -3\frac{1}{2})$, *e* $(+2\frac{1}{2}, -2, -1\frac{1}{4})$, *f* $(+2\frac{1}{2}, -2\frac{1}{2}, --1\frac{1}{4})$, $(g+3\frac{1}{2}, -2\frac{1}{2}, 0)$, *h* $(+3\frac{1}{2}, -2, 0)$. Draw the projections of a *bent plate connection* and in a section drawing show the true angle between its sides.

159. A cylindrical pipe, 2 feet in diameter, passes through the roofs shown in Fig. 14 at a point in the intersection of the roofs. The axis of the pipe is perpendicular to the line of intersection and is inclined 45° from the vertical. Find the true size of the hole cut out of the roofs for the pipe.

160. Obtain by development the true size and shape of the covering needed for a symmetrical dome with eight sides.

161. In Fig. 49 a portion of a locomotive boiler is shown. Make the top element of the slope sheet at an inclination of 30° to the horizontal, and show the true size of a steel plate to be used in making the slope sheet.

162. The sand-box on a locomotive stands partly on the slope

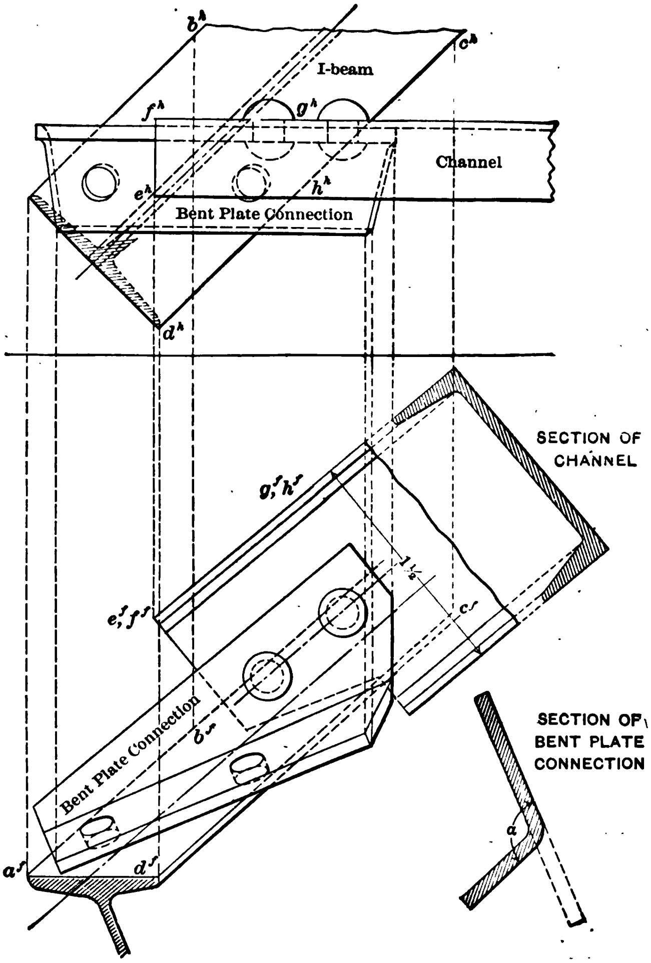
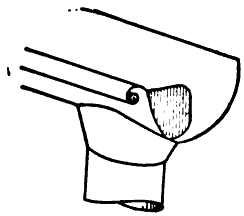

b^h
c^h
I-beam
f^h
g^h
Channel
e^h
h^h
Bent Plate Connection
d^h
SECTION OF CHANNEL
g^f, h^f
$1\frac{1}{2}$
e^f, f^f
c^f
Bent Plate Connection
b^f
SECTION OF BENT PLATE CONNECTION
a
a^f
d^f

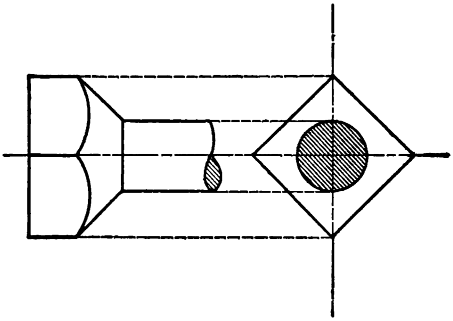
FIG. 48.

sheet and partly on the cylindrical portion. Find the shape of the bottom of the sand-box to fit the boiler drawn for Ex. 161.

163. Make the pattern for the conical portion of the eave-trough outlet shown in the figure.



164. A block of wood with a square cross-section has been turned in a lathe to the shape shown in the figure. Show the lines of intersection between the part with the square section and the conical portion.

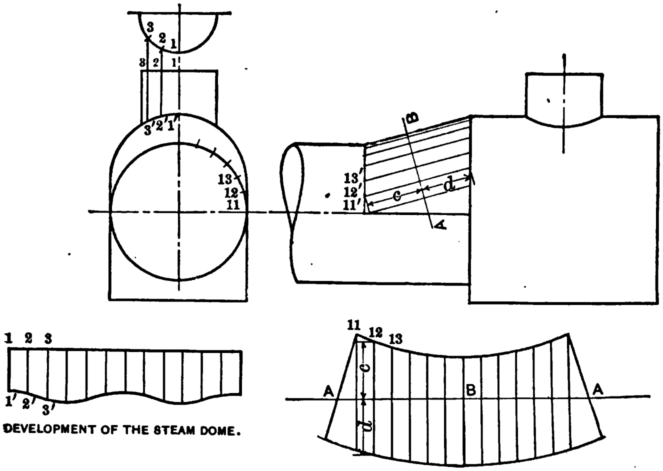


165. Make the patterns for the oil-can and grocer's scoop shown on page 160.

166. The stack of a boiler is supported by gusset-plates as shown in Fig. 50. Make the patterns for the gusset-plates.

167. Find the intersection of a sphere with a cylinder whose axis does not pass through the center of the sphere.

168. Find the intersection of the cylindrical ceiling of a corridor with the hemispherical ceiling of a vault.

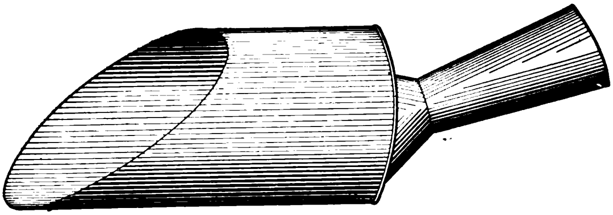


DEVELOPMENT OF THE STEAM DOME.

DEVELOPMENT OF THE SLOPE SHEET.

FIG. 49.

169. A semicircular wire loop, $1\frac{1}{2}$ inches in diameter, rotates on a vertical axis supported at the points $a\ (-1, -\frac{1}{2}, -\frac{1}{4})$ and $b\ (-1, -\frac{1}{2}, -1\frac{3}{4})$ at the ends of its vertical diameter. A ball, $\frac{1}{2}$ inch in diameter, is attached to the end of a horizontal supporting arm and revolves about the point $c\ (-2\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{2})$. The distance from the center of the ball to the center of revolution is 1 inch. Will the ball meet the wire loop?



170. A metal shade for an electric lamp is made up of a hemisphere and half of a circular cylinder. The axis of the cylinder passes through the center of the sphere. Draw the intersection of the two surfaces and a pattern to be used in cutting out the metal to make the shade.

171. Find the intersection of a hexagonal prism with an oblique plane. Draw the development of the part of the prism above the line of intersection.

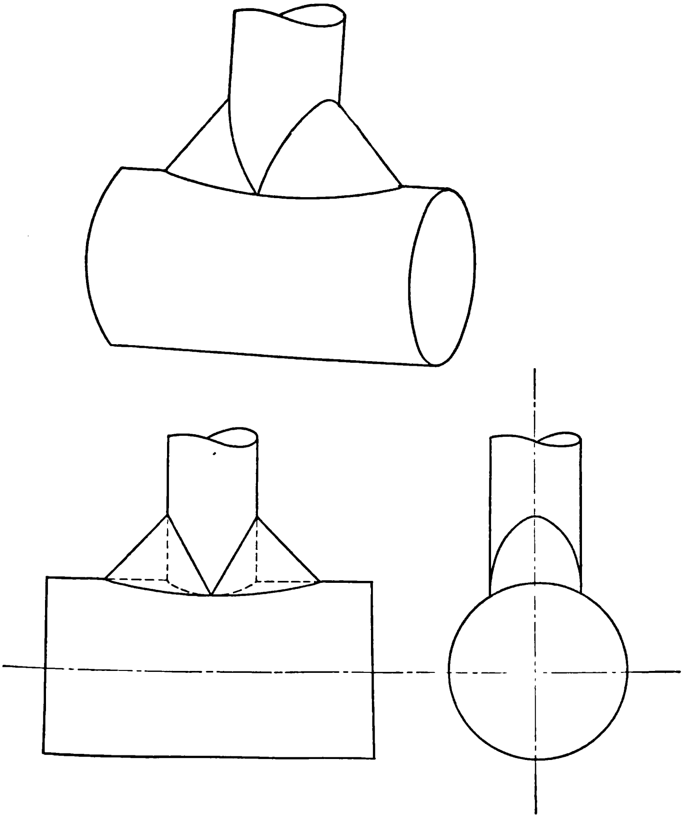
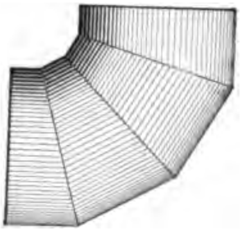


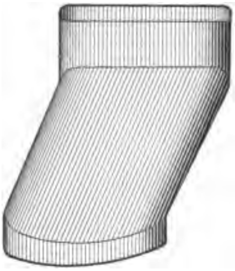
Fig. 50.

172. Make a pattern for one of the sections of the stovepipe elbow shown in the figure.



173. In Fig. 51 a buttress in a sloping wall is shown. The dimensions e and g are 10 feet and 8 feet respectively. Assume other suitable dimensions, and make all the patterns a stone-cutter will need for making the top stone of the buttress.

174. Make the patterns for the furnace-setter's "offset boot" shown in the figure. The section of the top portion is oval and of the lower portion circular.



175. Find the size of the hole that must be cut in the roof of the shed in Ex. 137 to allow a tight belt to run between the pulleys q and r.

176. Make the patterns for the ash-chute head shown in Fig. 52.

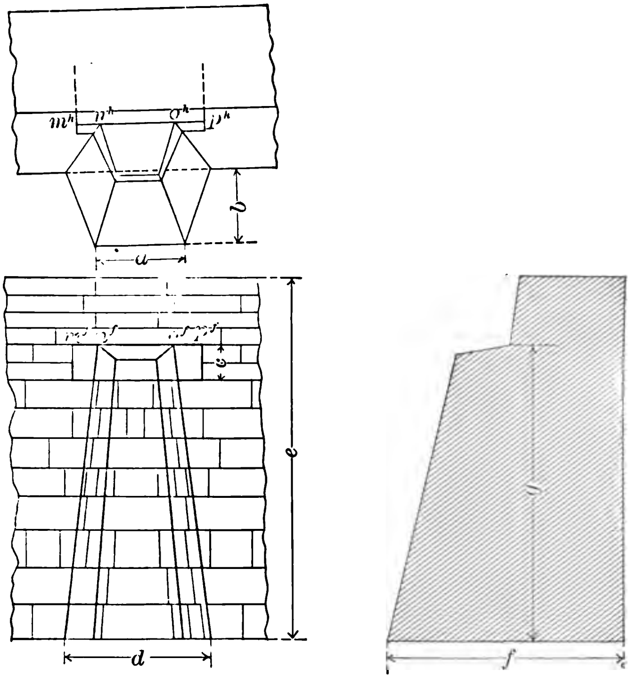
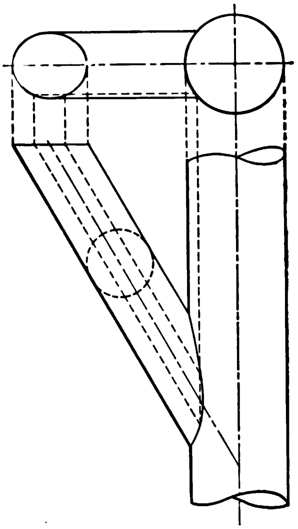
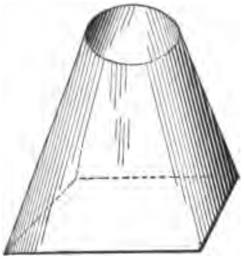


FIG. 51.

177. Develop a portion of the vertical pipe in the accompanying figure to show the true size of the hole to be cut so that the smaller pipe, inclined 30° to the vertical, may be joined to it. Both pipes have circular sections.



178. Make the patterns for the transition connection between a square duct and a circular pipe.



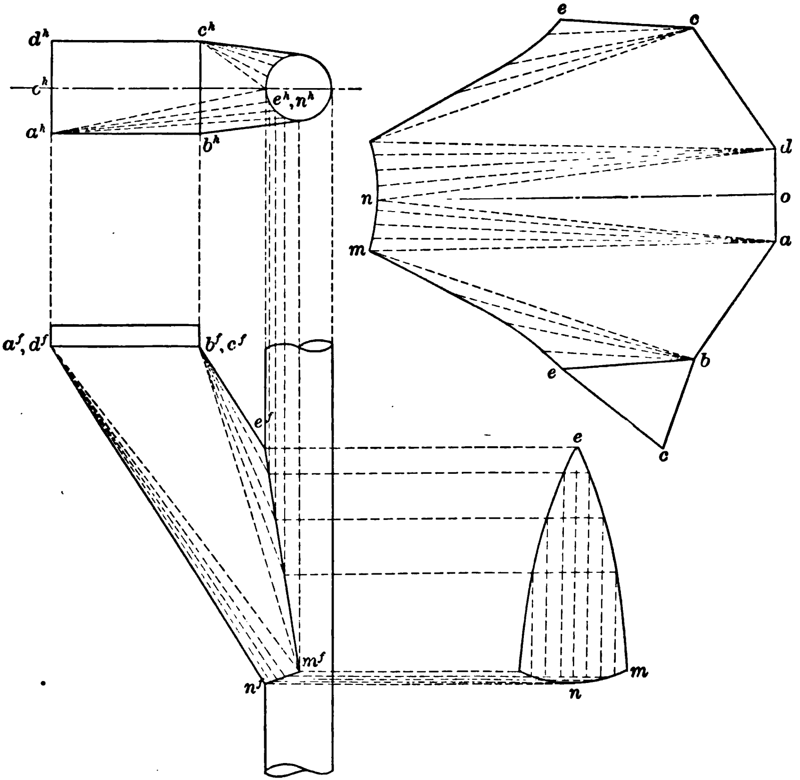


FIG. 52.

CHAPTER VI

SHADES AND SHADOWS

94. A person who understands descriptive geometry can usually get a correct conception of an object from the orthographic projections of its edges or other lines of its contour. A proper conception of the form and relations of the different parts of some objects requires, however, more careful study of the projections than is desirable for practical purposes. For this reason drawings are sometimes made to show an effect similar to that produced by the shadows from illumination. This effect is a great assistance also in making drawings plain to persons who are unfamiliar with the methods of orthographic projection.

The subject of **shades and shadows** treats of the application of the methods of descriptive geometry to produce the effect of illumination, which gives a more real appearance to the projections of an object.

If an opaque body is placed near a source of light, part of the surface will be bright and the remainder will be dark. A portion of the light from the luminous body will be intercepted and a portion of the space behind the body will be in darkness. This dark space is called the **shadow** of the body, and the line on its surface separating the bright side from the dark is the **shade line.**

95. In Fig. 53 a cone is shown near a vertical plane *P*. Parallel rays of light are represented passing through the points *a*, *b*, *c*, *d*, and *v*. The intersections of these rays with the plane determine the **shadow** of the cone on the plane. The **shade line** is found by joining points on the surface where rays are tangent.

96. Unless it is otherwise specified, the rays of light are to be represented by parallel lines. By the usual convention in

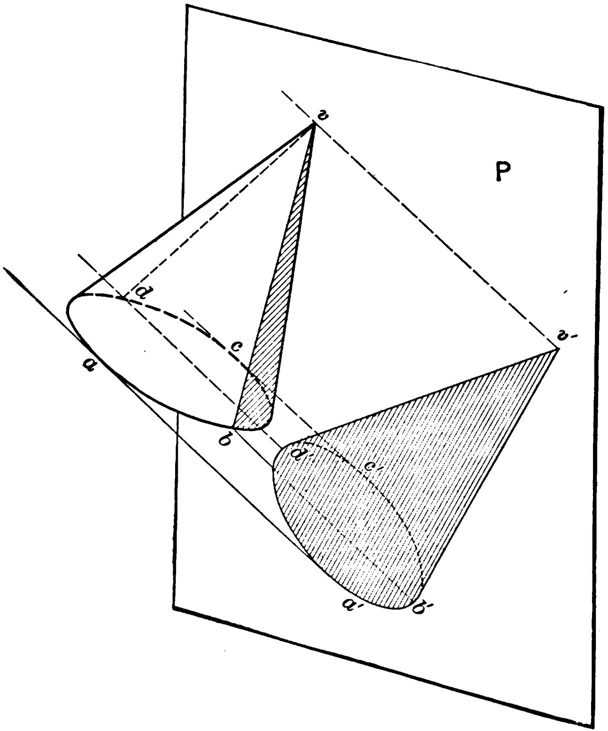


FIG. 53.

practical drafting the rays are taken as coming over the left shoulder, so that the horizontal and front projections of a ray make respectively angles of 45° and 315° with the X axis.

97. The method for constructing the shadow of a cone on the horizontal plane through its base is shown in Fig. 54. The projections of a ray through the vertex, v, are first drawn, and the intersection of this ray with the plane of the base is found at the point marked by the projections v^1 and v^2. Lines are drawn in the horizontal projection tangent to the base and through v^1. These are the limiting lines of the shadow and determine at the points of tangency the shade lines $c\,v$ and $d\,v$.

Fig. 55 shows the shadow of a "circular cap" on a cylindrical column. Scarcely any explanation is needed. Rays are drawn through points on the "cap" and their intersections with the surface of the column are easily found. The horizontal projections of the rays determine the points of intersection. The vertical shade lines at t' and j' are determined by tangent rays.

98. The shadow of an ellipsoid on a horizontal plane, P, is shown in Fig. 56. The *line of shade* is made use of to determine the shadow. Obviously it is the line of contact with the ellipsoid of a "cylinder" of rays tangent to the surface. This line of contact is, of course, an ellipse. In the figure two projections of an ellipsoid are shown. If a vertical auxiliary plane, A', is drawn parallel to the direction of the rays and through the center, o, of the ellipsoid, it will cut an ellipse from the ellipsoid and two elements tangent to the ellipse from the "cylinder" of rays. These points of tangency will be the highest and lowest points in the ellipse of shade. If the plane A' is revolved about a vertical axis through o so that it is parallel to the front plane, the section cut from the ellipsoid by the auxiliary plane will be made to coincide with the front projection. A ray drawn through the center o intersects the horizontal plane P at the point marked by the projections o' and o^2. After revolution with the auxiliary plane this ray is shown by the projections $o^h\,o_1$ and $o'o_2$. Draw the front projections of rays parallel to $o'o_2$ tangent

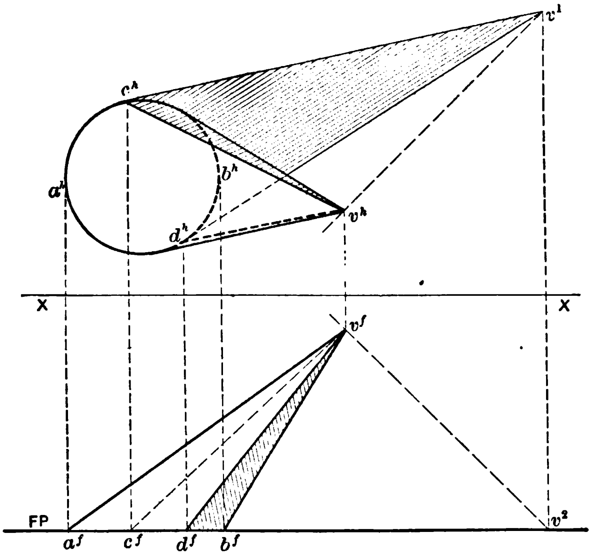


FIG. 54.

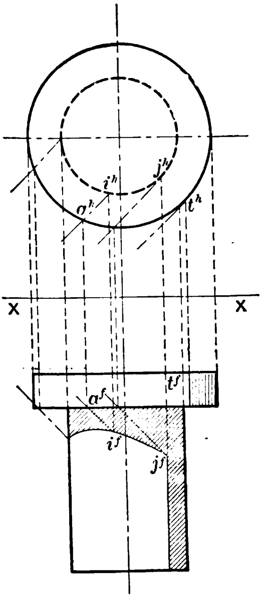


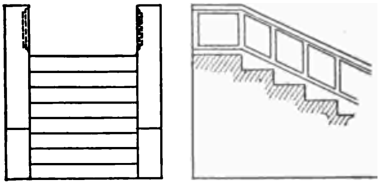
FIG. 55.

to the revolved section of the ellipsoid at a_1^f and b_1^f. When the plane A' is revolved back to its former position these tangent points are at a and b. In the front projection, then, a^f b^f is the major axis of the ellipse of shade. In the horizontal projection of the ellipsoid, a^h b^h is the minor axis of the ellipse of shade, and e^h f^h perpendicular to a^h b^h is the major axis.

The shadow of the ellipsoid on the horizontal plane, P, is the shadow cast by a section of the ellipsoid included by the ellipse of shade. To find this shadow determine the intersections with the plane, P, of rays through the points o, a b, e, and f. These points determine the major and minor axes, a' b' and e' f', of the elliptical shadow.

EXERCISES

179. Find the shadow of an ellipsoid on a vertical plane.
180. Find the shadow of a sphere, (*a*) on a plane below and parallel to the horizontal plane; (*b*) on an oblique plane.
181. Draw the shadow of a dormer window on a roof.
182. Draw the shadow of a chimney on a roof.
183. The stair ramp shown in the figure is parallel to the slope of the steps. The inclination of the stairway is 1 vertical to 2 horizontal. Construct the shadow of the ramp on the steps.



184. Draw the "cap" and column in Fig. 55 with the axis horizontal instead of vertical. Show the shade and shadow.
185. Make a simple drawing of a section of an engine cylinder through the center line. Represent the whole piston in the cylinder by showing the shadow on the inside of the cylinder of the portion that projects beyond the section.

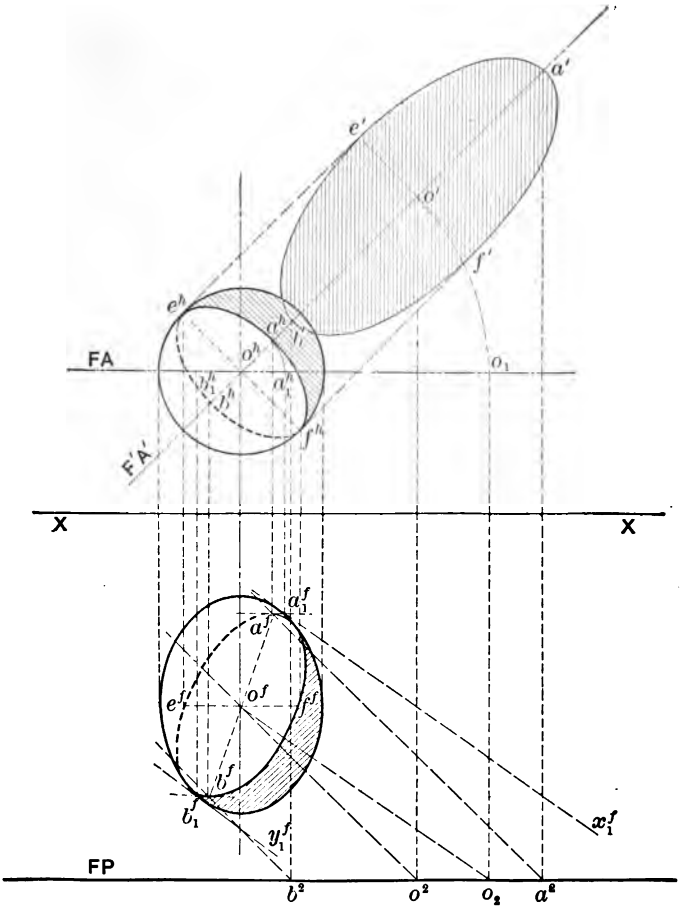

a'
e'
o'
f'
e^h
a^h
y_1^h
o^h
o_1
FA
b_1^h
b^h
a_1^h
f^h
F'A'
X
X
a_1^f
a^f
e^f
o^f
f^f
b^f
b_1^f
y_1^f
x_1^f
FP
b^2
o^2
o_2
a^2

FIG. 56.

CHAPTER VII

WARPED SURFACES

99. Warped surfaces are distinguished from plane surfaces and surfaces of single curvature (cones, cylinders, and convolutes) by a different position of the elements with respect to each other. In all the surfaces that have been discussed, a plane could always be passed through at least two of the consecutive elements. This property was very serviceable for the solution of problems. In warped surfaces, however, no two consecutive elements lie in the same plane; that is, **lines drawn through any two consecutive positions of the straight line generating them are neither parallel nor intersecting.** Examples of warped surfaces are shown in Figs. 57*a*, 57*b*, and 58.

A **warped surface,** therefore, may be generated by a straight line moving so that it **always touches two given lines and remains parallel to a given plane.*** The fixed lines which the moving line touches are called the **directrices** and the plane is called the **plane director.**

100. The **hyperbolic paraboloid** is a warped surface with a plane director and two straight line directrices which are not in the same plane. It takes its name from the fact that curved sections made by planes cutting the surface are either hyperbolas or parabolas (cf. Fig. 59). As the directrices approach parallelism the surface approaches a plane as a limiting surface. This surface is of some practical importance, as it is used in masonry construction, and the cow-catcher of a locomotive (Fig. 60) is usually of this form.

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* There is a great variety of warped surfaces, all differing in their mode of generation and properties; but this explanation is sufficient for the surfaces that will be discussed.

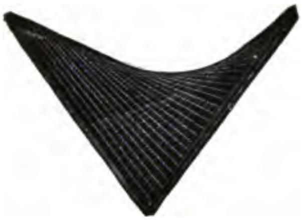


Fig. 57*a*.

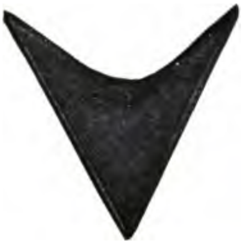


Fig. 57*b*.

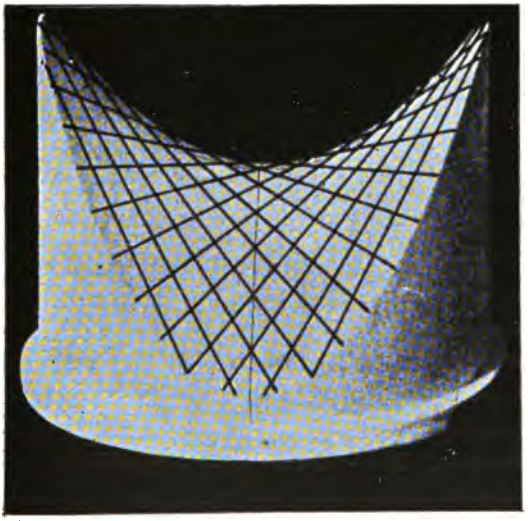


Fig. 58.

In a hyperbolic paraboloid, any plane parallel to a plane director will cut each directrix in a point, and the line joining these two points will be an element of the surface. The elements may be regarded as lying in a series of planes which are parallel to the plane director and dividing the two directrices proportionally. If, then, any two straight lines not in the same plane are divided into proportional parts, the straight lines joining the corresponding points of division are elements of the surface of the hyperbolic paraboloid.

101. The characteristic properties of this surface can be most conveniently investigated by reference to a pictorial drawing showing the directrices and the plane director. In Fig. 61 we shall take for simplicity of representation, the horizontal plane marked H for the *plane director*. The lines $a\,c$ and $b\,d$ shall be the *directrices*. From points on these lines the elements of the surface, $a\,b$ and $c\,d$, are drawn parallel to the plane director, H. For the necessary construction, draw through the directrix $a\,c$ a vertical plane, V_1, parallel to $b\,d$ and cutting the plane director in the line $x_1\,x_1$. Then draw another vertical plane, V_2, parallel to V_1 through $b\,d$, cutting the plane H in the line $x_2\,x_2$. The line $c\,f$ is drawn perpendicular to $x_1\,x_1$, and $d\,g$ is drawn perpendicular to $x_2\,x_2$; therefore $f\,g$ will be parallel and equal to $c\,d$. Through any point, i, on $a\,c$ draw $i\,j$ parallel to the plane director, H, and cutting $b\,d$ in j; then $i\,j$ is another element of the surface. The line $j\,k$ is drawn perpendicular to $x_2\,x_2$, and $i\,l$ perpendicular to $x_1\,x_1$; then $l\,k$ will be parallel and equal to $i\,j$. Also,

$$\frac{a\,i}{i\,c}=\frac{a\,l}{l\,f} \quad \text{and} \quad \frac{b\,j}{j\,d}=\frac{b\,k}{k\,g}.$$

Therefore $\frac{a\,l}{l\,f}=\frac{b\,k}{k\,g}$, and it follows that $l\,k$ and $f\,g$ cut $a\,b$ in the same point r. Now draw a plane parallel to V_1, cutting $a\,b$ in any point n, then its intersection with H will be parallel to $x_1\,x_1$, and will cut $l\,k$ in o, $f\,g$ in m, and $\frac{n\,o}{n\,m}=\frac{l\,i}{f\,c}$; also $o\,q$ is parallel and

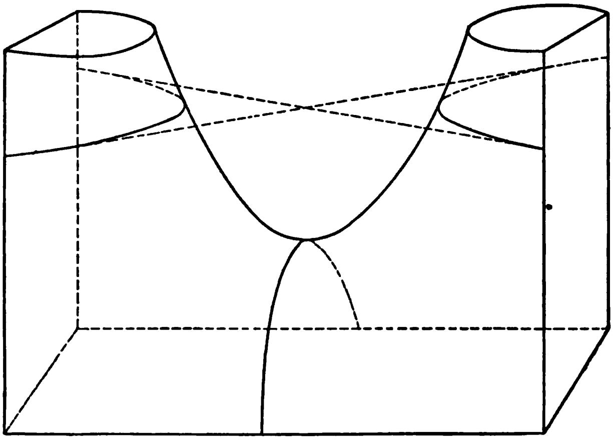


FIG. 59.

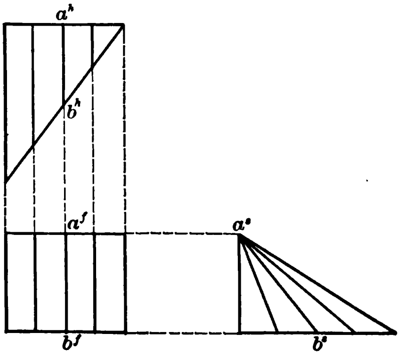


FIG. 60.

equal to $l\,i$ and $m\,p$ is parallel and equal to $f\,c$. Therefore $\frac{n\,o}{n\,m}=\frac{o\,q}{m\,p}$, which proves $n\,q\,p$ is a straight line, intersecting the three elements $a\,b$, $i\,j$, and $c\,d$, and dividing these elements proportionally, so that

$$\frac{a\,n}{n\,b}=\frac{i\,q}{q\,j}=\frac{c\,p}{p\,d}.$$

If the elements $a\,b$ and $c\,d$ are taken as directrices and the vertical plane V_1 as a plane director, the surface which is obtained will be identical with that having $a\,c$ and $b\,d$ as directrices and the plane H as its plane director. The elements $a\,b$, $i\,j$, etc., are called elements of the **first generation,** and those of the other set, as $a\,c$, $n\,p$, etc., which are parallel to the vertical plane, V_1, are called elements of the **second generation.** We have shown, then, that every element of either generation intersects all those of the other.

This surface is *doubly ruled;* that is, it has *two sets* of straight-line elements, and through any point on the surface two straight-line elements can be always drawn. Each element of one generation intersects no element of its own set, but meets all the elements of the other set. The lines on the surface in Fig. 58 show this double ruling very plainly.

102. Elements of the surface of a hyperbolic paraboloid are represented in Figs. 62 and 63. Two straight lines, $a\,b$ and $c\,d$ of *definite* length, and not in the same plane, are divided into the same number of equal parts. The lines joining corresponding points of division are elements of the surface. An element marked $x\,y$ is shown in each figure. Observe that the difference between the two figures is merely that the same points of division on $a\,b$ and $c\,d$ are joined up differently. In Fig. 62 the method of joining the points of division is the same as shown in the pictorial drawing in Fig. 61.

103. If the directrices of a *warped surface* in general are not limited in length, it is necessary to have given the directrices of both "systems of ruling" or the directrices and plane director of

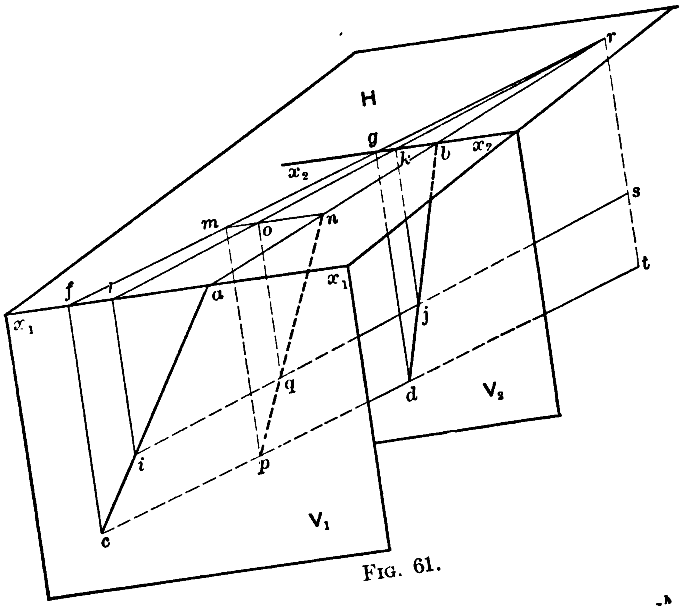


FIG. 61.

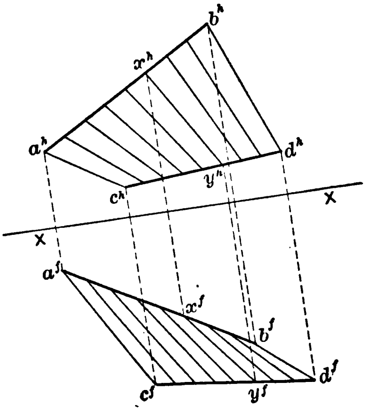


FIG. 62.

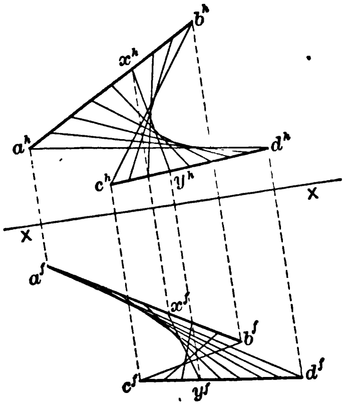


FIG. 63.

one system. This last case is illustrated in Fig. 64. The lines $q\,r$ and $s\,t$ are the directrices, and the plane director is given by its traces $H\,P$ and $F\,P$. To draw an element of the surface, then, some point as e on the directrix $s\,t$ is assumed. Through this point draw two lines $a_1\,b_1$ and $c_1\,d_1$ parallel to $a\,b$ and $c\,d$, which are any two lines in the plane director. The intersection of the plane of the two lines through e with the directrix $q\,r$ is a point in the required element of the surface. This intersection can be found very simply without finding the traces of the plane of the lines $a_1\,b_1$ and $c_1\,d_1$. First pass through the line $q\,r$ a plane perpendicular to the front plane and find the intersections of the lines $a_1\,b_1$ and $c_1\,d_1$ with this plane. The intersection of $a_1\,b_1$ with this plane has, of course, its front projection in the front trace at g', and a projecting line determines the horizontal projection. Similarly, $c_1\,d_1$ intersects the plane of $q\,r$ at f, and the plane of the lines $a_1\,b_1$ and $c_1\,d_1$ intersects the directrix, $q\,r$, at i. The line $e\,i$ is in a plane parallel to the plane director and touches the two directrices. It is therefore an element of the surface through an assumed point e. By this process any number of elements can be drawn.

104. If one projection of a point on any warped surface is known, another projection may be located by drawing through the given projection a line perpendicular to the plane of projection in which the given projection is represented. Through this perpendicular line pass a plane which will intersect the elements of the surface in points which, when joined, will give a line on the surface. The intersection of this line with the other projection of the perpendicular line is the required projection of the point.

105. The **hyperboloid of revolution** is a warped surface generated by a straight line revolving about an axis not lying in the same plane with it. Usually a vertical line is taken for the axis. Projections of this surface showing a number of elements are represented in Fig. 65. The method for constructing this surface is shown in Fig. 66. The axis $a\,b$ is vertical. An element of the

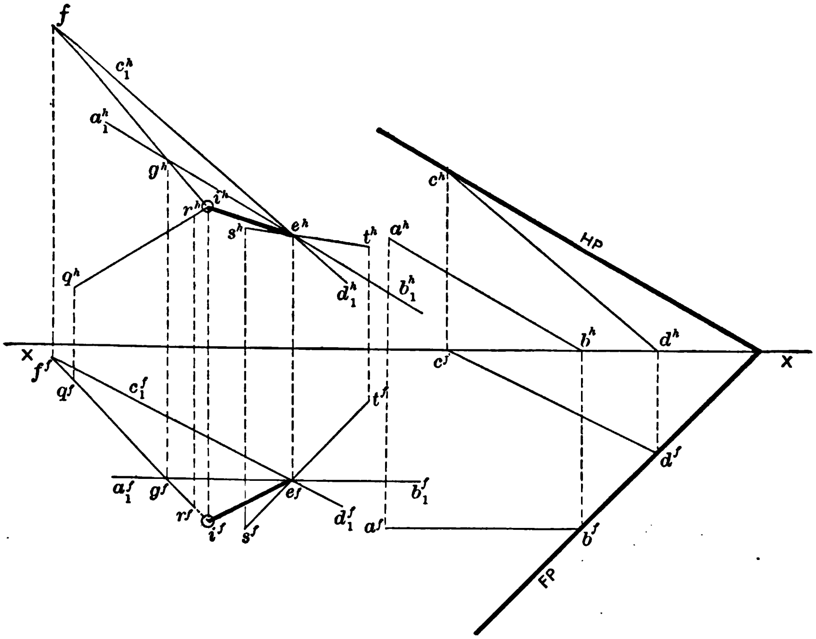

f
c₁ʰ
a₁ʰ
gʰ
rʰ iʰ
sʰ
eʰ
tʰ
aʰ
cʰ
HP
qʰ
d₁ʰ
b₁ʰ
bʰ
dʰ
X
X
fᶠ
qᶠ
c₁ᶠ
cᶠ
tᶠ
dᶠ
a₁ᶠ
gᶠ
eᶠ
b₁ᶠ
rᶠ
iᶠ
sᶠ
d₁ᶠ
aᶠ
bᶠ
FP

FIG. 64.

surface, as *m n*, is called a **generatrix,** because the surface can be generated by revolving this line about the axis. Each point in *m n* as it revolves about the axis describes a horizontal circle. The true radius of each circle is seen in the horizontal projection. Thus the point *r* on the generatrix nearest the axis describes the circle of which the diameter is shown in the front projection of the surface as $u^f r^f v^f$. This smallest circle described by a point in the generatrix is called the *circle of the gorge.* The diameter of the circle described by a point *p* is $q_1{}^f q_2{}^f$, which is equal to $2 \times b^h p^h$.

The hyperboloid of revolution is a *doubly ruled* surface, as it has two *sets* of straight-line elements. One element of each set can be drawn through any point on its surface. An element of one generation can be produced to intersect all those of the other generation, for the simple reason that they lie upon the same surface. If, then, any three elements of either generation are taken as directrices, any element of the other generation may be taken as a generatrix which, when moving, will produce the same surface.

The method for drawing a plane tangent to a hyperboloid of revolution is shown in Fig. 67. The plane, *P*, shown in the figure, is tangent to the surface at the point *c*. If only one projection of *c* is given, as say c^h, draw an element of the surface *d e* through *c* and find c^f by projecting to the front projection of *d e*. One of the other set of elements passing through the point is shown by *f g*. The two elements determine the plane tangent to the surface at the point *c*.

106. A helicoid or "**screw surface**" is a surface generated by a straight line moving so that it is always touching two helical directrices lying upon concentric cylinders. In Figs. 68 and 69 two cylinders of revolution are shown. On each is a portion of a helix. The straight line *m n* generates the surface of the helicoid in both figures. The difference is merely in the slope of the generating line. When both of the directing helices of a helicoid have the *same pitch* it is called a helicoid of *uniform pitch.* If the *pitches are not the same* the surface is called a helicoid of *varying pitch.*

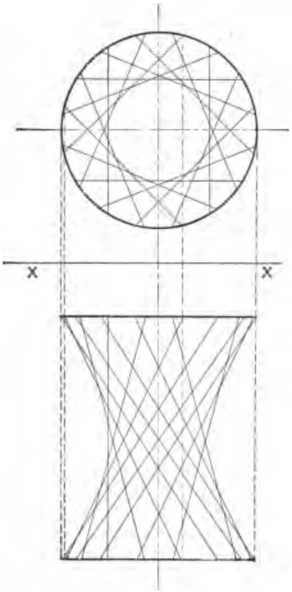

X
X

FIG. 65.

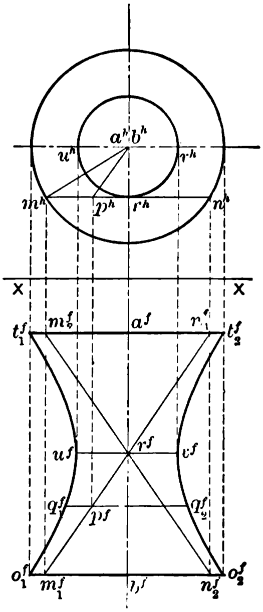

a^h b^h
u^h
v^h
m^h
p^h
r^h
n^h
X
X
t_1^f
m_0^f
a^f
r_1^f
t_2^f
u^f
r^f
v^f
q_1^f
p^f
q_2^f
o_1^f
m_1^f
b^f
n_2^f
o_2^f

FIG. 66.

The two examples of helicoids of uniform pitch in Figs. 68 and 69 differ only in the relative position of the moving or generating line. In both cases this line moves uniformly around the vertical axis, while, at the same time, all the points on it move uniformly in a direction parallel to the axis. In either figure the point *b* in the generating line *m n* will describe the helix *a b c d* lying on the inner cylinder, and the points *m* and *n* will describe helices of the same pitch lying on the outer cylinder. These helices traced by *m* and *n* are marked respectively *e f g* and *h i j*. The generating line *m n* will always lie in a plane tangent to the inner cylinder, and will intersect the vertical element at the point of tangency at a constant angle.

In the helicoid of Fig. 68 the generating line, *m n*, is always perpendicular to the vertical elements of the inner cylinder, and is called a *right helicoid*. The helicoid of Fig. 69 has the generating line, *m n*, inclined to the vertical elements of the inner cylinder, and is called an *oblique helicoid*.

When the elements of a helicoid are perpendicular to the axis, the surface is the same as that of a square-threaded screw (Fig. 70), and when the angle is less it is that of a V-threaded screw (Fig. 71). For the V-thread of the United States standard screw the angle between the elements and the axis is 60°.

107. For a satisfactory representation of a helicoid it is not necessary to draw a large number of elements of the surface. Usually it is shown better by drawing a few elements for a small portion of the surface. In Fig. 72 an oblique helicoid is shown resembling the one in Fig. 69. The small portion of the outline that is represented is easily recognized as like that in the groove of an auger or a twist-drill.

If on this surface it is required to locate, say, the front projection of a point when the horizontal projection is given, draw the horizontal projection of the element of the surface at the point. The required front projection of the point is then found by projecting to the front projection of the element. If only the front projection of a point is known, the horizontal projection

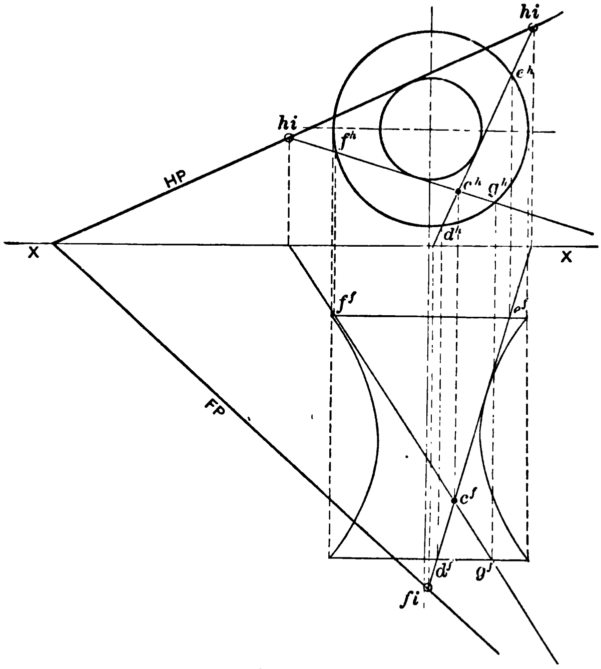


Fig 67.

is determined by drawing a line through the front projection parallel to the horizontal plane. Any plane passed through this line will cut the elements in points determining a curve. The horizontal projection of this curve will intersect the corresponding projection of the line at a point which is the required projection.

If the surface of the helicoid in the last figure is cut by a vertical plane *through* the axis and perpendicular to the front plane, the section cut out will be like the shaded drawing at the right-hand side of the figure. Half of the curve shown in the section is marked $s_1\, x_1\, y_1$. This figure is easily constructed by drawing elements of the surface. The points in which the elements are intersected by the cutting plane determine the curve of the section. Thus an element $i\, j$ intersects the cutting plane at x. This point is located on the curve in the section by projecting horizontally from x^f and laying off from the axis the length $o_1\, x_1$ equal to $o^h\, x^h$. The other points needed to determine the curve are found in the same way. Such a section cut from the surface by a plane through the axis is called a *meridian section.*

108. A plane is drawn tangent to the surface of a helicoid at a given point by drawing an element through the given point and a tangent to the helix lying in the surface and passing through the given point. These two lines determine the required tangent plane. As the surface is warped the plane will not be tangent throughout the length of an element, and will not contain, therefore, a tangent to the base at its intersection with an element passing through the point.

109. The intersection of the helicoid with any given plane is found by passing horizontal auxiliary planes through it. These planes will cut from the surface "spirals" *equal to the base,* and from the given plane horizontal lines. The intersections of these lines will give points on the required line of intersection. The projections of the different "spirals" are not needed if a curve of the base is made on a transparent sheet.

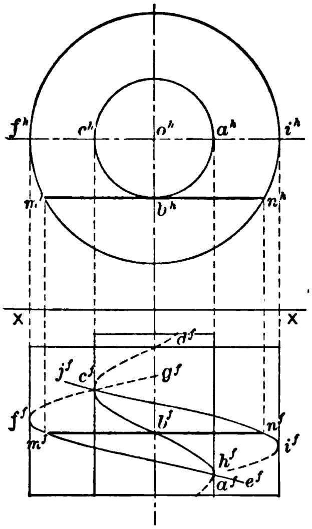


FIG. 68.

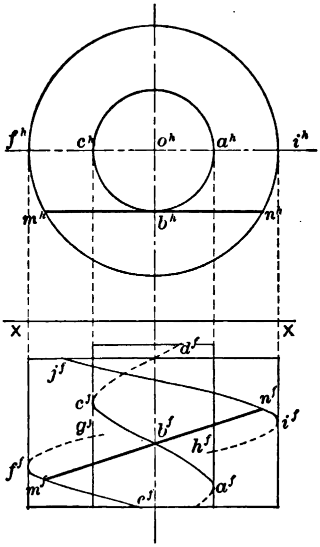


FIG. 69.

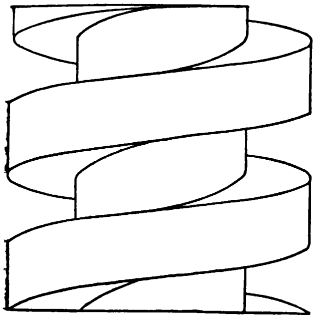


FIG. 70.

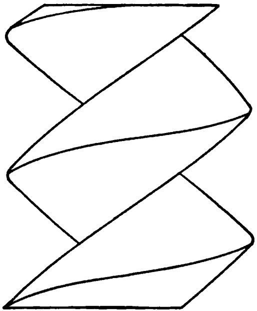


FIG. 71.

EXERCISES

186. Locate a point on the surface of a hyperbolic paraboloid given by two limited lines. Draw a tangent plane through this point.

187. Three oblique lines are given as belonging to one set of elements of a hyperbolic paraboloid. Find three elements of the other set.

188. Show several elements of the surface of a hyperbolic paraboloid given by two directrices and a plane director.

189. Draw the projections of a hyperboloid of revolution and assume a point on the surface. Draw a plane tangent to the surface at this point.

190. Assume the projections of an oblique helicoid. Locate a point on the surface and draw a tangent plane at this point.

191. Take the projections of the helicoid constructed for the preceding exercise and draw a meridian section and three transverse sections.

192. Draw the projections of (*a*) a square-threaded screw, (*b*) a V-threaded screw.

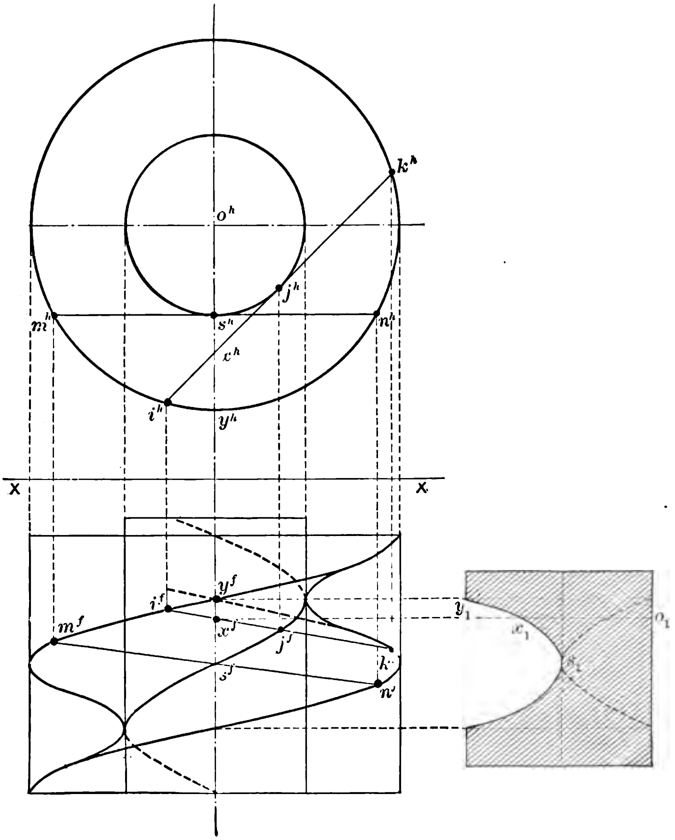


FIG. 72.

CHAPTER VIII

PERSPECTIVE

110. Perspective is the art of representing objects as they appear to the eye. The principles are not difficult for a student who understands the methods of orthographic projection.

The significance of perspective drawing can be explained best by showing in what way it differs from orthographic projection. In perspective the eye of the observer is at a finite distance from the object. In orthographic projection the views represent the object as seen when the eye is infinitely distant. By the perspective method, then, the lines drawn from points on the object to the eye, converge and intersect at the point of sight.

111. A perspective drawing represents an object as seen through a vertical plane which is assumed to be transparent. In Fig. 73 the eye at S, the object (a square pyramid $v\,a\,b\,c$), and the perspective projection $v^p\ a^p\ b^p\ c^p$ are plainly shown. The light solid lines in the figure represent the rays from the corners of the object to the eye. The straight lines joining the intersections of these rays with the vertical plane form the outline of the object. If this perspective picture is shaded and colored, it will exactly represent the object as seen from the viewpoint.

The vertical plane on which the perspective drawing is made is called the **picture plane,** and the position of the eye is called the **point of sight.**

The simplest construction of a perspective drawing is obtained by the use of two projections of the point of sight, together with the two corresponding projections of the object. The method is illustrated in Fig. 74. The light solid lines show the horizontal and front projections of a rectangular card, held nearly horizontally. Two

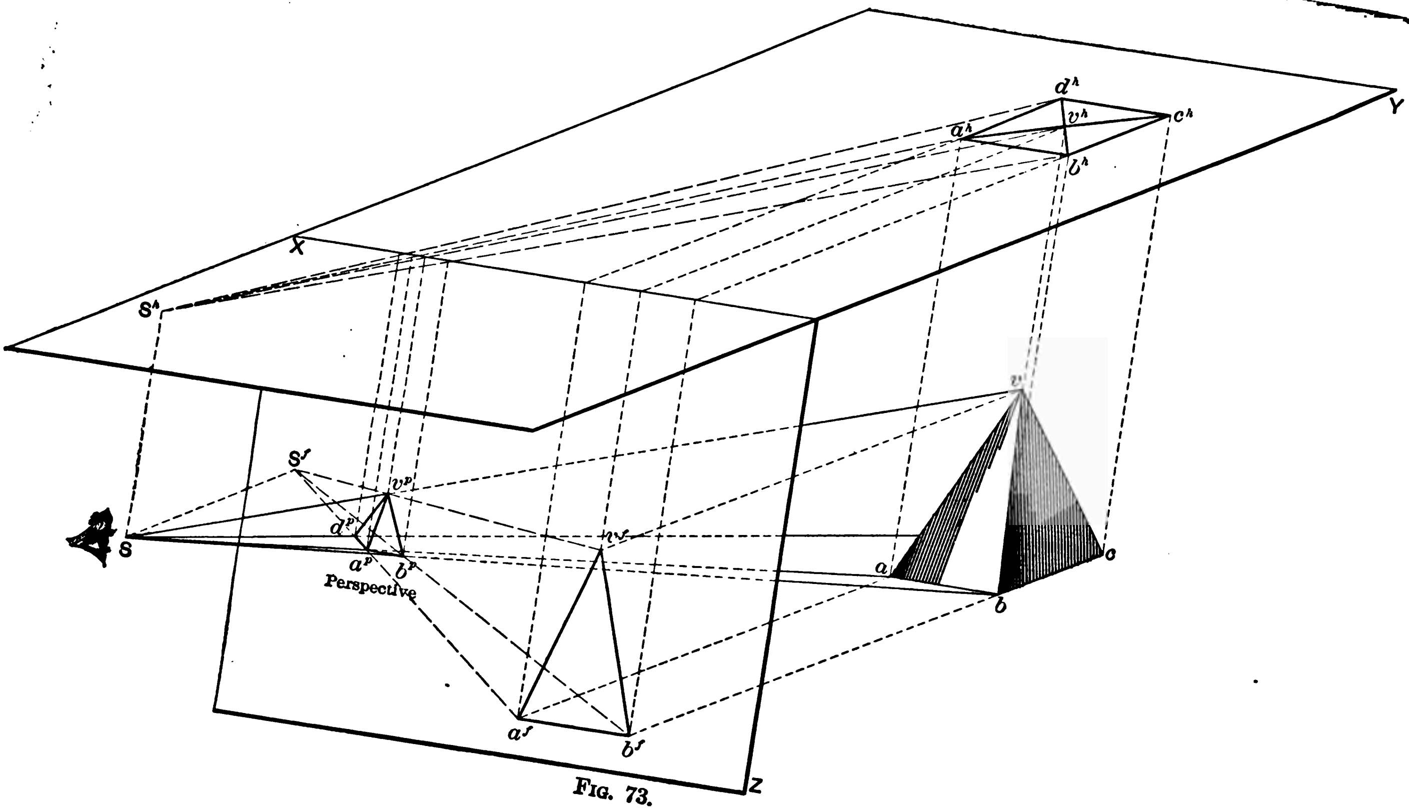


FIG. 73.

corners are marked *a* and *b*. The projections of the point of sight are S^h and S^f. Dotted lines are drawn joining the corners of the card in the horizontal projection with S^h and joining the corners in the front projection with S^f. These are respectively the horizontal and front projections of rays from the corners to the point of sight. From the points where the *horizontal* projections of these rays intersect the *X* axis, lines are drawn perpendicular to the axis to intersect the corresponding front projections of the rays. The corners of the perspective projection are thus determined, and by joining these corners the outline of the card shown by the heavy solid lines is obtained. Fig. 75 shows a perspective drawing of a screen with three "blades" made by this method. The points *a*, *b*, and *c* are marked to make the construction plainer. Usually when representing an object in perspective, the horizontal plane on which the object rests is shown. In the figure the line marked *GL* is the front trace of the horizontal plane through the bottom edges of the screen. If a house is to be represented in a drawing, the level of the ground is represented by a horizontal plane. The front trace is then very properly called the **ground line.** This name is given, however, to the front trace of any horizontal plane on which an object is imagined to be placed.

112. A picture may be made very simply by the method described, if there are not many details to be shown. In practical drawing, however, this method is not often used. The difficulty is that for a picture of suitable proportions the horizontal projection of the point of sight, S^h, must be usually located so far from the *X* axis that it is beyond the limits of the drawing-board. Also for an object with many details the number of construction lines becomes so great as to be very confusing. It is then often difficult to decide which points are to be joined. For these reasons, then, an abridged method of perspective drawing is commonly used.

In Fig. 76 a line *a b* is shown by its projections $a^h\ b^h$ and $a^f\ b^f$. This line is parallel to the horizontal plane and makes an angle of 45° with the front plane. By the method that has been just explained, its perspective projection is found at $a^f\ b^p$. The other

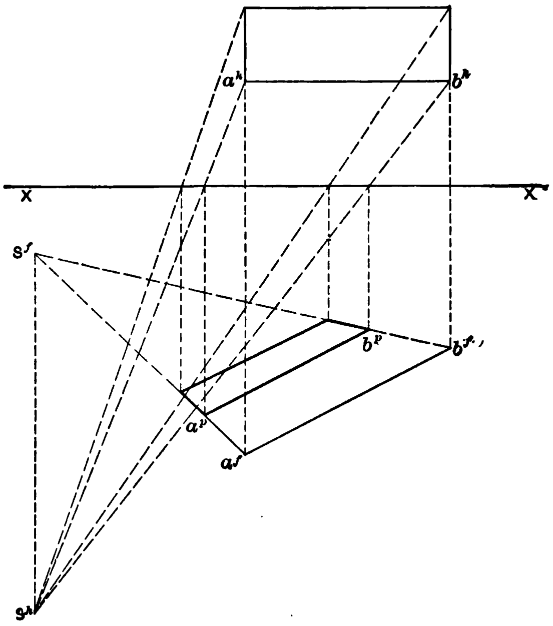


Fig. 74.

general method for constructing perspective drawings may be illustrated with this figure. The line $a\,b$ is produced indefinitely, as shown in both projections. A projecting line from S^h to the end of the line through $a\,b$ must be represented parallel to it and making an angle of 45° with the X axis. This projecting line meets the X axis at u; and the perspective of a point on ab at an infinite distance is at M. The perspective of any point on the line produced through a and b is, therefore, between a^f and M In this construction, observe that $S^h t = tu = S^f M$. Horizontal lines making the same angle with the front plane, but sloping in the opposite direction, would converge toward a similar point located at an equal distance to the left of S. It follows, then, that **all horizontal lines at 45° to the picture plane (front plane) converge in perspective toward points on either side of S^f and at a distance from this projection equal to that of the point of sight from the picture plane.** Since the location of the point M depends only on the *direction* of $a\,b$, and *not* on its *position*, any horizontal 45° line in the drawing will converge toward M. This point is called the **measuring-point.**

Some dimension lines are shown in Fig. 76 to show how this point can be used for laying off distances. For example, the perspective projection of the point b could be found by laying off the distance from b^h to the X axis on the left-hand side of b^f, and b^p would be found at the intersection of this line with the projecting line joining b^f and S^f. In the same way the perspective of a point c is found by laying off the distance from c^h to the X axis on the left-hand side of c^f and locating c^p at the intersection of this line with $c^f\ S^f$. If the measuring-point M had been located on the left-hand side of S^f, then in these same cases the distances mentioned would be laid off on the right-hand side of the front projections.

This method of constructing perspective drawings with the help of measuring-points is shown also in Fig. 77. A perspective drawing of a cube with circles inscribed in its sides is illustrated. The measuring-point M is located by making $S^f M$ equal to the distance from the point of sight to the picture plane (S^h to the

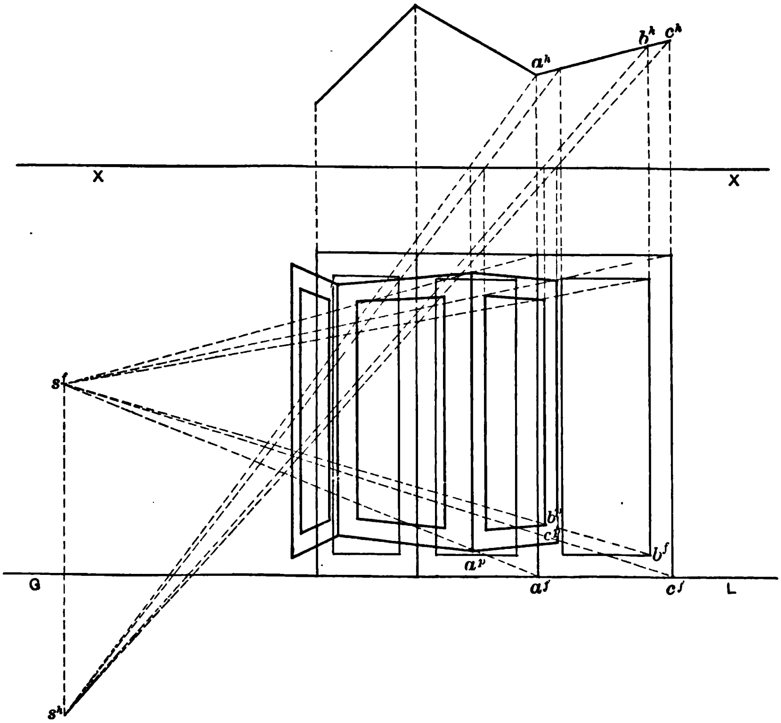


FIG. 75.

X axis). Observe that there are a number of parallel lines in a cube. The construction can be simplified if we notice that all parallel lines must converge or "vanish" at the same point. Every set of parallel lines has then a **vanishing-point**. The lines which are perpendicular to the picture plane have, of course, their vanishing-point at S'; and horizontal 45° lines vanish at M. For any drawing the vanishing-points, the measuring-points, and the front projection of S must be in a line parallel to the X axis.

The judicious use of measuring- and vanishing-points saves much labor in making perspective drawings. In Fig. 77 the vanishing-points of the horizontal edges of the cube are at V_1 and V_2. Each is located by drawing an edge for which the direction has been determined, to intersect the line through S' parallel to the X axis.

113. The perspective drawing of the cube illustrates also a very satisfactory method for drawing the perspective projections of circles. Points in the perspective drawing of the circles are found by locating diagonals of the circumscribing squares formed by the edges of the cube. Other lines are also drawn parallel to the sides of the same square through the intersections of the diagonals with the circle. The intersections of these straight lines locate usually enough points to determine the perspective of the circle. The construction for two points a and b of the circle are illustrated in the figure.

114. **Perspective Distortion.**—The principles of perspective are not difficult to apply in a mechanical way by those who have no artistic training; but distorted results are obtained from absolutely correct applications when absurd conditions are assumed. If, for example, a large house is represented with the point of sight about twenty feet from the front of the house, obviously a poor result is obtained. Nevertheless for such a case the principles can be applied as well as to any other.

A perspective drawing should show the object as it appears to the eye. If is important, therefore, that the best viewpoint is

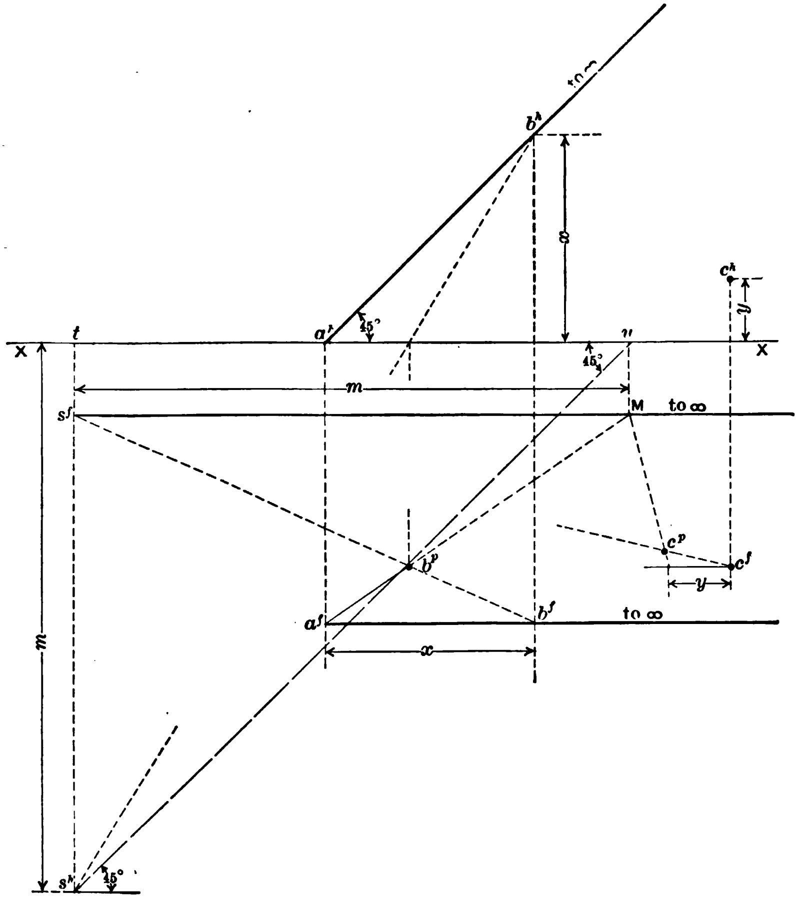


FIG. 76.

obtained; and care in selecting the viewpoint is as essential as a knowledge of the rules. If a house about forty feet high is to be sketched, the point of sight should be taken about eighty feet from the picture plane. A good rule to follow is to make this distance about twice the greatest dimension. When large objects are to be represented, the most satisfactory results are obtained usually when the point of sight is taken nearly in front of the object. It is preferable, then, that the *plan* drawing should be shown inclined to the picture plane.

115. Perspective Sketches from "Working" Drawings.—A very profitable application of the methods of perspective drawing is found in making free-hand sketches from "working" or "shop" drawings shown in orthographic projection. The "working" drawing represents certain information about an object by a *collection* of views. Several views are necessary to represent the object completely. In perspective drawing the same information is shown in a single sketch. When beginning a perspective drawing which is to be made from any projection drawings, it is necessary to acquire a thorough knowledge of the form and details of the object. The work, otherwise, cannot be done intelligently and rapidly. The perspective sketches for most objects should be commenced by drawing in perspective the edges of either circumscribed or inscribed solids,—usually square prisms. Most machines and architectural forms are easily treated in this way. The principal edges of the object should be then grouped into three systems, corresponding to three non-parallel edges of the prism. One of these will be represented by vertical lines, and two others must be shown with their proper convergence. After the principal edges of the object have been drawn, the other lines are very easily represented. When dealing with complicated forms it is absolutely necessary to follow some definite system to obtain results showing reasonable accuracy.

Even with simple drawings some care should be exercised in selecting the point of sight. It should be taken so that the details which are considered most important will appear in the

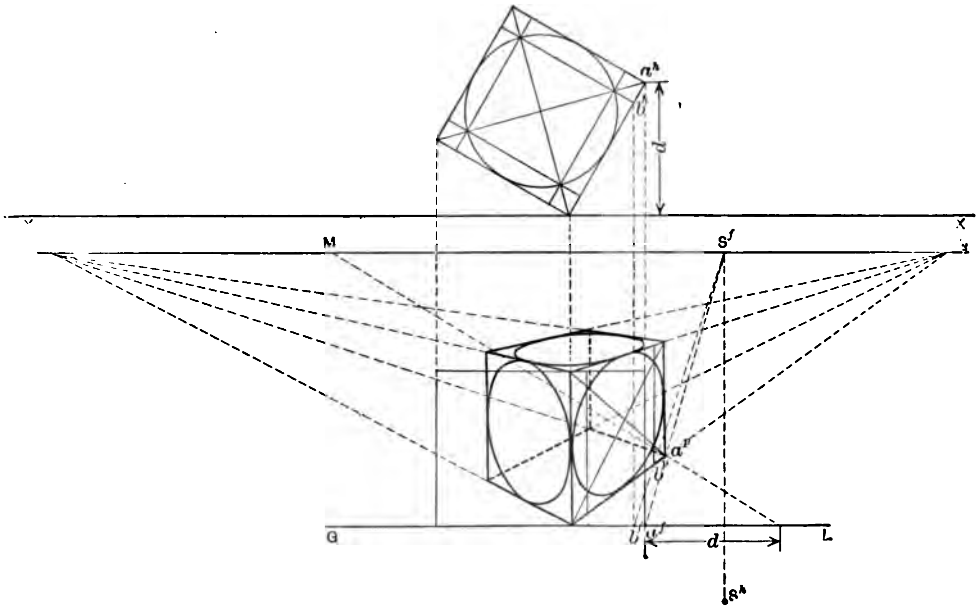
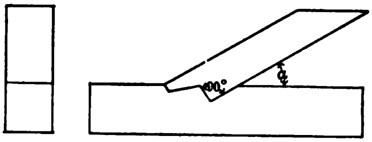


FIG. 77.

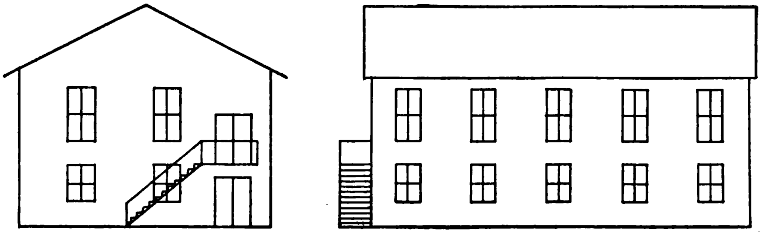
perspective sketch as plainly as possible. A drawing is of little value in which the important parts are crowded so that they are not clearly shown.

PRACTICAL EXERCISES

193. Make a perspective drawing of a screen with four "blades," using for the construction measuring and vanishing points.
194. Make a perspective drawing of the end connection of a truss as shown in the figure.



195. Make a perspective drawing of a simple wooden bridge.
196. Draw a cottage in perspective.
197. Show in perspective the flight of steps and the ramp in the figure in Ex. 183.
198. Make a perspective drawing of a locomotive boiler.
199. Make a perspective drawing of the mill building shown in the figure. Show also a tall chimney behind the building.



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